Ecological Economics 147 (2018) 123–133

Contents lists available at ScienceDirect

Ecological Economics

journal homepage: www.elsevier.com/locate/ecolecon

Analysis

# Do Mature Economies Grow Exponentially?[☆]

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## ARTICLE INFO

*Keywords:*
Per capita gross domestic product
Economic growth
Exponential growth
Linear growth
Time series analysis
Post-growth
Degrowth
Secular stagnation
Theories of economic growth

## ABSTRACT

In academic discussions and in public debate, economic growth is commonly presumed to be exponential. Economic theories model growth in an exponential manner and central policy institutions regard growth rates of 2–3% to be normal, also implying growth to be exponential. In this paper we investigate empirically whether economic growth is indeed exponential by estimating autoregressive integrated moving average time series models based on gross domestic product data for 18 mature economies from 1960 to 2013. Our findings cast doubts on whether these commonly discussed economic growth paths reflect the economic reality: only two out of 18 mature countries depict exponential growth rates above these levels. Five have lower exponential growth and the development of eleven countries exhibit rather linear growth. Additionally, we show that prominent theories of economic growth assume growth to be exponential and that a more heterogeneous set of theories is needed to explain different patterns of growth across time and space.

## 1. Introduction

The idea that economic growth is exponential is deeply rooted within economic discourses. In economic theories, growth is commonly depicted as a (constant) *fraction* of the level of production and therefore tends to be regarded as exponential.[1] Public economic debates also refer to the *rate* of economic growth, also implying exponential growth. The assumption of a *normal* growth rate of 2% or more also finds its way into political agendas. The USA's central bank, the Federal Reserve (FED), discusses "normal" growth rates between 2 and 3%, and the 2017 US government is even more optimistic, targeting 4% annual economic growth and "assuming" 3%.[2] This presumption is not only followed by governments, but also international institutions, such as the World Bank, who predicted a continuous growth in gross domestic product (GDP) of 2.5% in industrialized countries (Shaw et al., 2001) or the International Energy Agency which assumes the GDP in North America and Europe to grow at around 2.4% between 2000 and 2010 and around 2% between 2010 and 2030 (Birol, 2002). All these refer to the GDP and not gross domestic product per capita (GDPPC), which is at the core of this analysis. GDPPC is the more relevant variable with regard to economic theory for three reasons: in welfare economics, income per capita (and not overall income) is decisive. Concerning employment, economic growth per capita has to be equal to the rate of increases in labor productivity in order to prevent increasing unemployment. Further, several of the prominent theories of economic growth discussed below do not take into account changes in population. Therefore, we investigate the pattern of growth of GDPPC rather than GDP. Due to population growth, the formulated expectation of GDP growth of 2–3% translates into an expectation of GDPPC growth of about 1.3–2.3%.[3]

But do mature economies grow exponentially in reality? One emerging strand of literature argues that economic growth after World War Two has depicted a *linear* rather than an exponential pattern. The non-empirical literature on the issue includes Altvater (2006), and

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[1] Formally, exponential growth is given by $Y_t = Y_0 * (1 + r)^t$ in which $r$ is the growth rate. If $r$ is $> 0$ and stays constant over time, we observe exponential growth. For example, when the economy has the size $Y$ in period 0 and grows by amount $X$ which is the share $r$ of $Y$, then in the second period the economy has the size of $Y + X$. This means that — if $r$ stays constant — it grows by $r * (Y + X)$ in period two. So at a constant growth rate $r$, the absolute amount by which the economy grows becomes bigger each period.

[2] Central bank's figures taken from the FED's Monetary Policy Report (Federal Reserve System, 2014). Government figures taken from official White House press releases: the 4%-target is cited in Metro (2017, p. 106) and the "sustained, 3-percent economic growth"-assumption is being made in Mulvaney (2017).

[3] The population in the countries in our sample grew on average (unweighted) by 0.7% in the time frame of consideration between 1960 and 2013. When correcting the predictions of GDP growth of 2–3% for a population growth of 0.7%, this results in projected per-capita growth rates between 1.29% and 2.28% $\left(\text{formula:} \quad g_{\text{GDPPC}} = \frac{g_{\text{GDP}} - g_{\text{Population}}}{1 + g_{\text{Population}}}\right)$; population data from OECD database (OECD, 2017), accessed on 09.12.2017.

https://doi.org/10.1016/j.ecolecon.2018.01.011
Received 29 August 2017; Received in revised form 18 December 2017; Accepted 9 January 2018

Pollitt et al. (2010). The empirical literature on this pattern of growth is similarly scarce. Glötzl (2011), Seidl and Zahrnt (2014), Gordon (2012) and Reuter (2002) provide anecdotal evidence for Western countries, industrialized countries, the US, and Germany, respectively. Wibe and Carlen (2006) and Bourcarde and Herzmann (2006) use descriptive statistical tools to conclude that the majority of mature economies follow linear (and not exponential) growth paths. While the latter analyses and findings are interesting, their conclusions are not backed up by a rigorous application of econometric methodology.

Another recent strand of literature refers not to exponential or linear growth but to so-called *secular stagnation*. Especially after the recent economic crisis, authors in this debate do not only argue that growth rates have been declining over time but also give explanations for decreasing growth rates. Influential economists such as Lawrence Summers (2016, 2014b), Robert Gordon (2015) and Paul Krugman (2014) argue that the United States and other mature economies have entered a long phase of low growth. Explanations for such growth range from slow technological change to insufficient investments to satisfied markets (see Section 5.2).

The present study combines an empirical study of exponential vs. linear growth with the question, how this can be explained theoretically. In the empirical part, we differentiate between countries depicting linear growth, low exponential growth ( < 1.3%) and high exponential growth ( > 1.3%). The investigation goes beyond existing approaches of pure descriptive statistics and uses adequate econometric models — autoregressive integrated moving average time series models — which allow us to make credible quantitative statements about the long-term trends in the time series at hand. It contributes further to the literature by building upon most recent data and decidedly longer time series than the two empirical analyses mentioned above (see Sections 3 and 4). In the theoretical parts, it combines well-established theories of economic growth with less common approaches such as the new debate on secular stagnation and insights from the study of economic history.

The paper is structured as follows: Section 2 shows the ways in which the concept of exponential growth is implemented in prominent theories of economic growth. Sections 3 and 4 are devoted to the question of whether the common assumption that economic growth is exponential holds empirically. Section 5 poses the question, how theories of economic growth need to change in order to explain our empirical results — so that a wider range of patterns of growth can be covered. Section 6 concludes with suggestions for future economic theory.

## 2. Economic Growth Theories

Economic growth is a highly researched topic and well-represented in typical economics curricula. In the following, theories of economic growth patterns will be analyzed, including influential theories within mainstream, as well as heterodox, economics. The presentation of the theories is based on original contributions where one specific model is commonly referred to. Where a large number of similar models exists, a prominent textbook version is chosen.

### 2.1. Neoclassical Growth Theories

Solow presented a first growth model in line with neoclassical thought, a model subsequently followed by many others. In the following, four prominent neoclassical models are analyzed.

#### 2.1.1. The Solow Model

In the theory of Solow (1956), the growth rate of the capital stock $\left(\frac{\dot{k}}{k}\right)$ is determined by investments, which depend on savings ($sF[k]$) and the depreciation of the capital stock ($\delta k$):

$$\frac{\dot{k}}{k} = s\frac{F[k]}{k} - \delta \tag{1}$$

This model implies that capital per worker has decreasing marginal productivity. As the depreciation rate of the capital stock is a constant proportion, capital accumulation comes to an end when the marginal productivity of capital equals the depreciation rate. If Harrod-neutral (i.e. labor-augmenting) technological change is included, capital accumulation can continue to take place over time, as the marginal productivity of capital increases. In this scenario, the steady state rate of economic growth per capita is entirely determined by the speed of technological change $x$ (Barro and Sala-i-Martin, 2004):

$$g = x. \tag{2}$$

As $x$ is given exogenously, the model in principle allows for all types of patterns of growth per capita. At the same time, the model suggests exponential growth as there is no reason why $x$ should change over time. We are not aware of any attempts to incorporate explanations for a change in $x$ within these models.

#### 2.1.2. The Neoclassical Textbook Growth Model

While Solow assumed a certain savings rate and a certain investment behavior of firms, neoclassical growth models are based on the behavior of a representative household and a representative firm. Households maximize utility due to a utility function. Their savings depend on their preferences and the interest rate. Firms maximize profits. They invest until the marginal productivity of capital is equal to the real interest rate plus the rate of depreciation. Subsequently, in these models savings and investments are brought into equilibrium via the real interest rate (Barro and Sala-i-Martin, 2004). The equilibrium growth rate of the capital stock per capita is similar to the Solow-model. The change in capital stock is determined by the difference between output and consumption. Additionally, the capital needed for depreciation, and due to technological change, is subtracted:

$$\frac{\dot{\hat{k}}}{k} = \frac{f(\hat{k}) - \hat{c}}{\hat{k}} - (\delta + x). \tag{3}$$

However, the savings behavior, the state of technology and the depreciation rate have only level effects but no growth effects regarding production: 'A greater willingness to save or an improvement in the level of technology shows up in the long run as higher levels of capital and output per effective worker but in no change in the per capita growth rate' (Barro and Sala-i-Martin, 2004, p. 210). The growth of per capita income ($g$) depends — as in the Solow model — primarily on the rate of technological progress ($x$): 'the steady-state per capita growth rate equals the rate of technological progress, $x$, which is assumed to be exogenous' (p. 205).

$$g = x. \tag{4}$$

The neoclassical textbook model therefore leads to the conclusion on the nature of economic growth, that as long as technological change is assumed to be of a constant *rate*, growth per capita is exponential.

#### 2.1.3. Endogenous Growth I: The AK-model

Endogenous growth models have the advantage that they include an explanation of the central determinant of growth in the model — compared to earlier theories in which the determinant was most of the time exogenous to the model. In the AK-model, capital has constant instead of diminishing marginal returns, due to the fact that capital is understood in broader terms, including human capital. Production in this model depends on the technological state ($A$) and the amount of capital ($K$): $Y = AK$. Growth of capital $\left(\frac{\dot{K}}{K}\right)$ and income $\left(\frac{\dot{Y}}{Y} \text{ or } g\right)$ depend, without technological change, on the size of net investments, which depend on the savings rate ($s$) and the depreciation rate (Aghion and Howitt, 2009):

$$g = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = sA - \delta. \tag{5}$$

Assuming constant population, the rate of economic growth equals the rate of economic growth per capita. Therefore, similarly to the neoclassical models, the AK model predicts exponential growth per capita as long as the exogenous parameters remain constant. Decisive in this model are the constant returns to capital (designated by the fact that $K$ has the exponent 1) and a constant savings rate. If these parameters are constant, economic growth is exponential. Equally similar to the previous models, if these parameters in the AK model change over time, the result can be non-exponential growth.

#### 2.1.4. Endogenous Growth II: Imperfect Competition

Endogenous growth models with imperfect competition assume monopolistic competition in certain markets. Firms invest into new production methods or into the improvements of the existing production methods because they have a temporary patent on the new method which allows them to make profits (Aghion et al., 1998). The rate of technological change, which impacts the rate of economic growth, depends on the speed at which new intermediate goods are invented. This is determined by several factors: (1) the preferences of households concerning consumption and savings ($\theta$ represents the households' willingness to have different consumption levels over time and ρ stands for the time preference of the households), which will in turn influence the amount of resources put into developing new technologies (2) the speed of technological change is influenced by the price of inventing new technologies ($\eta$) and the size of the mark-up ($\alpha$) a firm can put on the new technology; (3) the current state of technology ($A$) and the amount of labor employed ($L$) influence the growth rate (Barro and Sala-i-Martin, 2004, Chapters 6 and 7):

$$\frac{\dot{Y}}{Y} = \frac{1}{\theta}\left(\frac{L}{\eta}A^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\alpha^{\frac{2}{1-\alpha}} - \rho\right). \tag{6}$$

Here, growth is again represented as a rate. A change in each of the factors mentioned therefore alters the rate of growth. Again, constant parameters lead to exponential growth, as well as exponential growth per capita, as the population size is assumed to stay constant in this model.

### 2.2. Heterodox Theories

In neoclassical frameworks, growth depends primarily on two aspects: 1) savings, which leads to investments and 2) technological change, which increases labor productivity. Heterodox growth theories also regard these factors as central, but analyze the underlying mechanisms very differently. Generally speaking, investments are not determined by savings but by other factors and technological change either depends on investments or is the result of market competition.

#### 2.2.1. Marxian Theories

In Marx's theory, firms buy labor (variable capital), materials and physical capital (constant capital) in order to manufacture products that they sell in a competitive market. Firms make profits through the exploitation of labor, i.e. workers are paid at their cost of reproduction that usually is below the value of their labor power (Harvey, 2006). As firms sell their products in a competitive market they are forced to reinvest profits into new production technologies in order to be able to offer products at a lower price. If they do not reinvest, they are outcompeted by rivals (Mandel, 1969). Investments are determined by the degree of competition ($a$) and the level of profits (π): $I = a\pi$. The profit rate $\left(r = \frac{\pi}{K}\right)$ results primarily from the bargaining process between capital and labor. The growth rate of the capital stock, which is equivalent to the growth rate of output ($g$) depends on these two factors (Hein, 2002):

$$g = \frac{I}{K} = ar. \tag{7}$$

Hence, economic growth is primarily determined by the degree of market concentration and the bargaining process between capital and labor. As long as $a$ and $r$ stay constant, $g$ is constant, so that exponential growth is predicted.

It is widely debated within Marxian economics whether $a$ and $r$ can really be assumed to stay constant over time (Sweezy, 1943). The discourse takes place under the term *tendency of the rate of profit to fall*.[4] On the one hand, it is argued that technological change leads to a change in the organic composition of capital (the ratio between variable and constant capital) and as a result decreases the profit rate over time (Harvey, 2010). The intuition behind this is that surplus is generated due to the exploitation of labor power. Therefore, ceteris paribus, using relatively less labor in the production process also implies less possibility to generate surplus and profits. On the other hand, if the surplus rate (the ratio between surplus and variable capital) increases, this tendency can be countervailed (Sweezy, 1943). The Marxian analysis therefore leaves it open as to whether economic growth per capita (assuming constant population) takes place at a certain rate (and hence is exponential) or declines. Hence, in contrast to neoclassical theories, the discussion around decreasing economic growth over time because of parameter change (in particular due to technological change) has a long history and a prominent position in Marxian discussions.

#### 2.2.2. Kaleckian Theories

Kalecki (1987) developed a growth theory that includes both Marxian and Keynesian analyses and concepts where investments are at the center of the argument and determine growth. Its effect depends on the wage share ($w$) and the consumption rate of profits ($q$) as both influence the multiplier effect of investments: $\Delta Y = \frac{\Delta I}{(1-w)(1-q)}$. According to Kalecki (1987) there are five central determinants of investments: (1) a firm's savings ($S$), which induces entrepreneurs to invest more, as they have more financial means; (2) an increase in a firm's profits $\frac{\Delta P}{\Delta t}$, which stimulates investments because production has become more profitable; (3) the growth of the capital stock $\frac{\Delta K}{\Delta t}$ which decreases investments because additional profits resulting from investments are lower for a higher level of existing capital; (4) an increase in actual production in the past $\frac{\Delta Y}{\Delta t}$ which induces investments because it implies a decrease in the relation between inventories and production; and (5) the investments may change due to various long running changes ($d$) in the economy (technological change, interest rates, company share earnings):

$$I_{t+1} = aS_t + b\frac{\Delta P}{\Delta t} - c\frac{\Delta K}{\Delta t} + e\frac{\Delta Y}{\Delta t} + d. \tag{8}$$

$a$, $b$, $c$ and $e$ are parameters for the strength of the determinants. Kalecki further argues that Eqs. (1)–(4) all primarily depend on the investments of the past and so current investments are a function of past investments, several behavioral parameters, the wage share, profit share and long-term changes ($d$): $I_{t+1} = aI_t - c\frac{\Delta K}{\Delta t} + \frac{1}{1-q}\left(b + \frac{e}{1-w}\right)\frac{\Delta I}{\Delta t} + d$.

Investments are the sum of a fraction of the past investment level ($aI_t$), the change of investments in the past multiplied by some constant $\left(\frac{1}{1-q}\left(b + \frac{e}{1-w}\right)\frac{\Delta I}{\Delta t}\right)$ and the change of the capital stock multiplied by some constant $\left(c\frac{\Delta K}{\Delta t}\right)$. Assuming constant behavioral parameters and a constant wage share, economic growth in Kalecki's theory is, therefore, also exponential. This is best illustrated by assuming that $a = 1$ and abstracting from the depreciation of the capital stock. Defining $f = \frac{1}{1-q}\left(b + \frac{e}{1-w}\right)$, we get

[4] Piketty (2014, p. 274) has found that the profit rate fell very slowly, from 4% to 3% over the last centuries.

**Table 1**
$R^2$ for exponential and linear models and difference between them. A positive difference indicates the preference of the linear fit. Time horizon: 1960–2013.

| Country | $R^2$ exp. fit | $R^2$ lin. fit | Diff. |
|---|---|---|---|
| Australia | 0.9910 | 0.9755 | −0.0155 |
| Austria | 0.9709 | 0.9951 | 0.0241 |
| Belgium | 0.9544 | 0.9886 | 0.0341 |
| Canada | 0.9597 | 0.9819 | 0.0222 |
| Denmark | 0.9491 | 0.9774 | 0.0283 |
| Finland | 0.9578 | 0.9728 | 0.0150 |
| France | 0.9329 | 0.9779 | 0.0449 |
| Italy | 0.8906 | 0.9526 | 0.0620 |
| Luxembourg | 0.9509 | 0.9356 | −0.0153 |
| Netherlands | 0.9686 | 0.9824 | 0.0138 |
| New Zealand | 0.9452 | 0.9246 | −0.0206 |
| Norway | 0.9526 | 0.9836 | 0.0310 |
| Portugal | 0.9198 | 0.9711 | 0.0513 |
| Spain | 0.9403 | 0.9699 | 0.0296 |
| Sweden | 0.9774 | 0.9711 | −0.0062 |
| Switzerland | 0.9566 | 0.9537 | −0.0029 |
| United Kingdom | 0.9702 | 0.9601 | −0.0101 |
| United States | 0.9760 | 0.9879 | 0.0119 |

$$I_{t+1} = I_t + (f - c)\frac{\Delta I}{\Delta t}. \tag{9}$$

As long as $f > c$, the growth of investments is therefore exponential in its nature. Assuming a constant capital coefficient and constant population, economic growth is therefore also exponential.[5] If parameters change over time on the other hand, growth can be of another nature.

### 2.2.3. Standard Keynesian Growth Model

Hein (2004) develops a standard Keynesian model of growth and distribution. In this model there are also solely savings out of profits, so the savings rate ($s$) is equal to the savings ratio out of profits ($s_\pi$) multiplied by the profit rate ($r$): $s = s_\pi r$. The growth rate of the capital stock is determined by *animal spirits* (human behavior in groups, $\alpha$) and by the investment reaction ($\beta$) to the profit rate ($r$): $g = \alpha + \beta r$. A steady-state growth rate exists, where the growth rate ($g$) is equal to the savings rate: $g = s$. Combining these equations, one gets the equilibrium growth rate:

$$g = s = \frac{s_\pi \alpha}{s_\pi - \beta}. \tag{10}$$

Once again, determining whether growth per capita is exponential depends on whether parameters change, in this case most importantly the savings rate, the profit rate and the behavior of entrepreneurs (animal spirits and reactions to a certain level of the profit rate).

### 2.3. Intermediate Results

This review of prominent theories of economic growth leads to two conclusions. First, most theories are set up in such a way that they imply exponential growth as the *normal* type of growth per capita. As long as the parameters in the models do not change, growth is exponential. In some cases, the models are explicitly designed to depict a steady state of economic growth, where all variables grow at constant rates. Second, in principle, all theories discussed can also display non-exponential patterns of per capita growth. However, this would imply changing parameters, which does not follow from the theories themselves. The only exception discussed here is the tendency of the rate of profit to fall in Marxian theories. It may be objected that there have been discussions on falling growth rates in other schools of economic thought and while this is true, to the best of our knowledge, such discussions have been of minor importance and have not been at the core of these theories. One indication is that they are not present in influential textbooks — whereas the tendency of the rate of profit to fall is included in any standard textbook on Marxian economics (e.g. Harvey, 2010; Heinrich, 2005; Sweezy and Marx, 1942).

## 3. Methods and Data

As we have seen, most prominent growth theories argue for an exponential growth pattern. This is represented by one or several ratios, be it the savings ratio, the speed of technological change, the profit rate or the potential productivity of investments. Accordingly, a Gross Domestic Product Per Capita (GDPPC) series underlying the exponential growth models covered so far (excluding the Marxian theory, which is not clear in this respect) can be represented within a simple regression framework by the following term:

$$\text{GDPPC}_t = b_0(1 + r)^t + \varepsilon_t,\ t = 0,\ \ldots, T - 1, \tag{11}$$

where $b_0$ denotes the starting value at time $t = 0$ of the series with $T$ observations, the rate $r$ describes the exponential growth of the series, and $\varepsilon_t$ is the error term for the observation at time $t$.

As an alternative hypothesis we test the exponential model against the simplest model of diminishing growth rates, which is a model of linear growth:

$$\text{GDPPC}_t = b_0 + (1 + r)t + \varepsilon_t,\ t = 0,\ \ldots, T - 1, \tag{12}$$

where $r$ now enters the equation in a linear fashion which corresponds to a constant increase of the GDPPC. In this constant growth model the growth *rate* converges towards zero as the time horizon becomes very long.

In order to examine these growth theories on an empirical basis that allow conclusions for economic decisions, we look at the economic development for a set of 18 mature economies from 1960 to 2013. In particular, we investigate the yearly real GDPPC series for the group of Western and Southern European countries and Western offshoots (as defined by Maddison, 2006, Appendix B). Germany is left out of the sample in order to avoid problems of aggregation during the period after re-unification in 1990. Luxembourg is added, which Maddison includes in the group of 'Small West European Countries' (Maddison, 2006, p. 179). A full overview of the selected countries can be found in Table 1. An ideal starting point for the analysis would be in 1950, as the effects of World War Two should have vanished by then. As Crafts and Toniolo (1996) argue, 'In five years at most, Europe recovered the ground lost relative to the highest prewar income levels. It is, thus, quite safe to place the end of the first phase of reconstruction and the beginning of a new era in the history of European economic growth in 1950' (Crafts and Toniolo, 1996, p. 3). However, since the data is only available since 1960, we start from there.

It has been argued that the second major reason for high growth rates after World War Two in Europe (including the majority of countries in our sample) was convergence. The US-American economy had introduced new production methods characterized by higher labor productivity over the previous decades, which European countries had not. Between the end of World War Two and the late 1960s the introduction of such technologies in European countries facilitated high growth rates (Eichengreen, 2008). In order to exclude this effect, we additionally execute the empirical investigation for the period 1970–2013.

Due to partly differing GDPPC measures, we analyze two data sets: World Bank GDPPC series in US$ with 2005 prices and Conference Board GDPPC series in US$ with 2014 prices. Since the results are very similar, we only report the results for the first data set mentioned above.[6] In this data set, GDPPC data for New Zealand and Switzerland are only available in the period 1970–2013.

[5] Note that assuming no population growth is the more conservative assumption for the argumentation laid out here because even without population growth the model predicts exponential economic growth. In the empirical analysis below, per capita GDP is therefore at the center of analysis.

[6] The data sets, non-reported results and the software code can be obtained from the authors upon request.

In a preliminary analysis we compare for each country the coefficients of determination $R^2$ between regressions of the GDPPC series on a linear and on an exponential time trend.[7] The first regression corresponds to Eq. (12) and the latter to Eq. (11). In order to obtain the optimal exponential growth rate within the linear regression framework and within standard software packages, the exponential growth model is estimated for a grid over 50 equidistant values between 0 and 0.06 for $\tilde{r}$ and the highest $R^2$ among all 50 models is selected.[8]

This procedure represents a rather descriptive approach of comparing the linear and exponential growth of the GDPPC. Making credible quantitative statements about the data generating process behind the time series is, however, only valid for stationary series. The simplest example is a so-called random walk process: such a non-stationary process often generates data sets which seem to have a trend behavior, even though the long-term behavior of a random walk is purely random and thus not predictable by any time trend. In order to be able to reasonably compare the adequacy of time series models with their potentially associated exponential or linear long-term trends, it is necessary to account for structural deviations from these trends described by unit roots and remaining autocorrelation in the time series. A common criterion to assess the adequacy of a time series model, the predictive ability, i.e. how well the model predicts observations out of the sample, will be described below. Note that these issues were not tackled by Wibe and Carlen (2006) and Bourcarde and Herzmann (2006).

We therefore apply the Box–Jenkins method (Box et al., 2008) to find suitable models for the series at hand. The first step is to determine the number of unit roots of the time series. Initiated by Nelson and Plosser (1982), there has been an extensive debate on whether macroeconomic time series are trend-stationary, i.e. stationary after removing a time trend, or follow a unit root process with a potential drift (see for instance Perron, 1989; Shin et al., 1992; Cuestas and Garratt, 2011). The latter view seems to be the more prominent in the literature as most authors apply unit root and co-integration techniques to macroeconomic time series such as the GDPPC. In order to check whether unit root methods are also required for the data set at hand, we conduct the augmented Dickey–Fuller test for both the original time series and the logarithm (log) of GDPPC series in each country, after removing their linear time trends. Note that removing a linear time trend from a log-transformed time series corresponds to the deletion of an exponential time trend of the original series and thus is appropriate if a GDPPC series follows an exponential growth pattern. In the augmented Dickey–Fuller test, a rejection of the null hypothesis suggests the absence of a unit root and therefore trend-stationarity. Likewise, we conduct the KPSS test for the same series, yet the null hypothesis for this test is trend-stationarity.

The results of these tests can be found in Table 3 in the Appendix. We find no strong evidence against a unit root and for trend-stationarity in both the original and the log transformed GDPPC in any of the countries except for Switzerland.[9] Thus, we generally assume that the series follow unit root processes with drift terms. We are aware of the heterogeneity of the countries and the weaknesses of the underlying tests, as pointed out in Cuestas and Garratt (2011). In particular, our simple time trend models might not capture true nonlinear patterns, e.g. as a result of shifts caused by policies. Nevertheless, as we are interested in the comparison between two specific growth patterns and as our test results are quite unambiguous, we are confident that the aggregate findings of the following analyses should be credible.

Assuming a unit root, the next step of the Box–Jenkins method is to determine the orders $p$ and $q$ of the autoregressive process and the moving average process in a suitable autoregressive integrated moving average (ARIMA) ($p$,1,$q$) model with drift/trend.[10] In order to do so, we use the *auto.arima* function of the *R* package *forecast* (Hyndman and Khandakar, 2008). More specifically, we choose among all candidate models with maximum lag orders $p = q = 3$, the most appropriate one with respect to the Akaike Information Criterion for finite sample sizes (AICc).[11] This procedure achieves an appropriate compromise between a lack-of-fit to the data and a too complex model. As we are assuming a unit root, we actually model the first differences of GDPPC series. Allowing for a potential exponential growth, this leads in the simplest case with $p = q = 0$ to the ARIMA(0,1,0) model with drift

$$\Delta \text{GDPPC}_t = \text{GDPPC}_t - \text{GDPPC}_{t-1} = \widetilde{b}_0(1+\tilde{r})^{t-1} + \tilde{\varepsilon}_t,\ t = 1,\ \dots, T-1, \tag{13}$$

where the errors $\tilde{\varepsilon}_t$ are assumed to be independent and identically distributed and $\widetilde{b}_0$ describes the difference between the first two observations which grows over time at the rate $\tilde{r}$. A constant and thus linear growth is given for $\tilde{r} = 0$ because then the difference between subsequent observations is always $\widetilde{b_0}$. We exploit exactly this property to decide for either a linear or an exponential growth model by using the following approach: first, for each of the 50 equidistant values between 0 and 0.06 for $\tilde{r}$, the most suitable ARIMA ($p$,1,$q$) model with drift with respect to the AICc is chosen.

In a second step we decide for the final model with a corresponding growth rate (which we subsequently refer to as the optimal model with optimal growth rate) by comparing the value of several selection criteria among all 50 ARIMA ($p$,1,$q$) models chosen in the first step.

To compare different time series models, often their predictive ability is assessed via the accuracy of pseudo-out-of-sample-forecasts. In those cases, the particular models are fitted to a subsample of the data, e.g. all observations except for the last 5 years. Then, the last data points are predicted based on the estimated model parameters and compared with the observed values which have been left out before. This procedure essentially tells us which model forecasts better at the end of the sample. However, we are aware of a potentially big influence of the recent financial crisis on the results if we solely compared forecast accuracy for the last years. Thus, in our second step of final model selection, we use the Akaike Information Criterion (AIC), which includes all observed data points and can thus be seen as a measure for predictive ability for the whole sample (Hyndman et al., 2008, chap. 7). Furthermore, we also base our selection of the optimal model on the AICc and the Bayesian Information Criterion (BIC). Despite the advantages of such model selection criteria, the results are still likely to depend on the time frame of the chosen sample. For this reason, we repeat the analysis excluding the recent financial crisis, i.e. considering only the years 1960–2007. As a further robustness check, we discard the first 10 years of the sampling period and thereby restrict our analysis to the years 1970–2013, in order to exclude the effect of convergence, as argued above.

## 4. Empirical Results

The results of the $R^2$ comparison between the best exponential fit (chosen among all 50 grid values for the growth rate $\tilde{r}$) and the linear fit can be found in Table 1. For the majority of countries, the linear model fits the data more accurately than the exponential one. In a

[7] Note that this approach is essentially equivalent to the comparison of the log-likelihood between a non-transformed and a log-transformed GDPPC series as done by Wibe and Carlen (2006) who used GDPPC data up to 2005.

[8] Clearly, all countries in our sample exhibit a positive growth within the time frame under consideration.

[9] We compared the p-values of the tests with a significance level of 5%.

[10] The second parameter of the model, commonly referred to as the order of integration $d$, is set to one because we assume one unit root in all series. Moreover, we assume the series to have a drift/trend in accordance to the research question whether the GDPPC in mature economies exhibits an exponential trend.

[11] We also used the Bayesian Information Criterion (BIC) for model selection in this step and obtained very similar results. For a comprehensive discussion on model selection criteria in time series models, see Hyndman et al. (2008, chap. 7).

**Table 2**
Optimal growth rates leading to the optimal ARIMA ($p$,1,$q$) model with drift with respect to different model selection criteria. A growth rate of (close to) zero implies a linear growth model. Time horizon: 1960–2013.

| Country | AIC | AICc | BIC |
|---|---|---|---|
| Australia | 0.0110 | 0.0110 | 0.0110 |
| Austria | 0.0000 | 0.0000 | 0.0025 |
| Belgium | 0.0000 | 0.0000 | 0.0000 |
| Canada | 0.0000 | 0.0000 | 0.0000 |
| Denmark | 0.0000 | 0.0000 | 0.0000 |
| Finland | 0.0000 | 0.0000 | 0.0000 |
| France | 0.0000 | 0.0000 | 0.0000 |
| Italy | 0.0600 | 0.0600 | 0.0000 |
| Luxembourg | 0.0037 | 0.0037 | 0.0037 |
| Netherlands | 0.0000 | 0.0000 | 0.0000 |
| New Zealand | 0.0135 | 0.0135 | 0.0135 |
| Norway | 0.0000 | 0.0000 | 0.0000 |
| Portugal | 0.0000 | 0.0000 | 0.0000 |
| Spain | 0.0000 | 0.0000 | 0.0000 |
| Sweden | 0.0025 | 0.0025 | 0.0025 |
| Switzerland | 0.0012 | 0.0012 | 0.0012 |
| United Kingdom | 0.0025 | 0.0025 | 0.0025 |
| United States | 0.0000 | 0.0000 | 0.0000 |

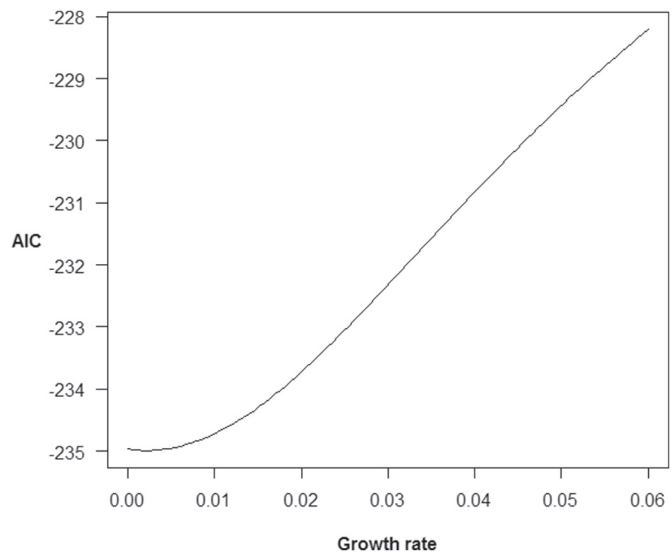

**Fig. 1.** AIC values for the ARIMA ($p$,1,$q$) models chosen for different values of the growth rate $\tilde{r}$, illustrated for Sweden. A lower AIC indicates a better predictive ability.

couple of countries, the evidence points in the opposite direction, but with minor differences between the models.

Table 2 depicts the optimal growth rate of the optimal ARIMA ($p$,1,$q$) model (with respect to the model selection criteria AIC, AICc and BIC after our two-step selection procedure) explaining the observed GDPPC series for each country.[12]

The linear growth model with $\tilde{r} = 0$ is preferred for 11 out of 18 countries with respect to the AIC. Among the remaining seven countries five countries depict low exponential growth rates below 1.3%. Note that some optimal growth rates are not equal but close to zero, indicating a quasi-linear growth. Such an example is Sweden which exhibits an optimal growth rate of only 0.25%. The behavior of the AIC over all grid values for the growth rate $\tilde{r}$ is shown in Fig. 1 for the case of Sweden. It can be seen that the lowest AIC at the growth rate 0.25% is very close to the AIC for the linear model with $\tilde{r} = 0$, whereas it becomes considerably larger for increasing values of $\tilde{r}$.[13]

The results are fairly robust for the chosen model selection criteria, with the only exception being Italy: here, the AIC and AICc indicate an exponential growth model with the largest growth rate $\tilde{r} = 0.06$ on our grid, whereas the BIC prefers the linear model. A closer look at the optimal ARIMA(1,1,3) model chosen by AIC and AICc shows a near unit root autoregressive coefficient of 0.9989 and an implausible negative coefficient for $\tilde{b}_0$ which would imply negative exponential growth.[14] In contrast, the BIC points towards an ARIMA (0,1,1) model with a positive linear trend. These unstable results suggest that the development of Italy's GDPPC is hardly predictable, at least within our model class and the limited database.

The financial crisis following 2007/08 led to a deep recession in the world economy and was an extraordinary economic event. Since, in our case, the model selection criteria are likelihood-based measures relying on the normal distribution, outliers strongly affect the choice of the optimal model and the corresponding optimal growth rate for differenced time series such as the ARIMA($p$,1,$q$) processes.[15] We therefore execute the analysis described above once again, excluding the years from 2008 on. As Table 4 in the Appendix indicates, the results for the same analysis based on a restricted sample (i.e. without the financial crisis) are quite different from the ones presented in Table 2. In four out of the 18 countries, a linear growth model with $\tilde{r} = 0$ is chosen. For another six countries the selected growth rate is below 1.3%, implying low exponential growth while models depicting high exponential growth are chosen for eight countries.

Another robustness check investigates the years 1970–2013 only. The results in Table 5 in the Appendix are qualitatively similar to the ones obtained for the entire sample period. For thirteen out of 17 countries (excluding Italy which again exhibits ambiguous results) a linear growth model is selected. Four countries are modeled best by exponential growth, three countries among them with a high growth rate larger than 1.3%.

In general, the variability of the results presented is partly explained by the short time horizon under consideration, i.e. the lack of information provided especially by the shortened time series in the analyses in which the first 10 and last 6 years are excluded, respectively. In particular, near-zero exponential growth is very difficult to distinguish from a linear growth pattern for the time horizons under consideration. However, the main question is whether mature countries are really characterized by commonly discussed growth rates above 1.3%. The linear growth model serves here as a simple benchmark.[16] It would be interesting to repeat the analyses in a couple of years when the time series horizon will allow for even more distinct conclusions.

While this analysis only includes linear and exponential (and no other) types of growth in ARIMA ($p$,1,$q$) models, other models might reflect the development of their yearly GDPPC more accurately for some of the countries subject to analysis. The approach employed here is the most conservative one, since expanding the analysis by allowing for more different kinds of patterns is likely to further decrease the number of economies that are best represented by exponential growth models.

Furthermore, modeling country-specific peculiarities of the time

[12] To check for robustness, the same analyses were carried out with GDP instead of GDPPC as the dependent variable. The results change as follows: for the years 1960–2013 (Table 2), the AIC selects an optimal value for $r$ larger than 2% for four countries compared to two when using GDPPC with its corresponding reference growth rate of 1.3% (see Section 1 for an explanation of reference growth rates). For the time frame 1960–2007 (Table 4), the number increases from seven to nine and for 1970–2013 (Table 5) the number is the same (four countries).

[13] Naturally, a quantitative comparison between a linear and an exponential growth model for a single country is in general not possible if the linear model is chosen, since there is no comparable best exponential growth model.

[14] For all other countries, the estimate for $\tilde{b}_0$ is positive and statistically significant at the 5%-level. The results are available upon request.

[15] More specifically, the squares of deviations from the mean enter into the estimation. Thus, an outlier in a differenced time series, which has little variation otherwise, has heavy weight on the results obtained. For a better technical understanding of the maximum likelihood estimation procedure, the reader is referred to background literature such as Fahrmeir et al. (2013).

[16] The intention of this work is neither to claim that the linear pattern is the true one nor to distinguish between very low exponential and linear growth.

series such as shifts caused by policies would require more elaborate analyses beyond the scope of this paper. Though, ARIMA models can already account for deviations from the assumed long-term linear or exponential time trend which is of the main interest here. While structural deviations, which might be caused by regularly occurring macroeconomic cycles of fixed length, are captured by the estimated ARIMA coefficients, nonstructural deviations are captured in the model errors.

## 5. Discussion: Theories on Non-exponential Patterns of Growth

The analysis in the previous section has shown that the concept of growth rates of about 2–3% in total GDP, or 1.3–2.3% growth per capita, do not fit the reality of growth in mature economies since the 1960s. Many countries had lower growth rates. But the insights go even one step further: the growth in many countries is better described by linear than by exponential growth. Contrary, we have shown in Section 2 that prominent growth theories suggest a pattern of exponential growth. The subsequent questions are therefore: how can non-exponential growth be explained theoretically? And what type of theories do we need for such explanations? Three interpretations of the empirical results are pointed out. First, the finding of linear growth in many cases calls for the development of linear growth theories. Second, the finding of linear growth could also be the result of (secular) stagnation in recent years. Third, it could be due to changing patterns of growth in economic history. These interpretations do not necessarily contradict each other but call for the development and employment of a plural set of theories of economic growth to explain diverse patterns of growth.

### 5.1. Interpretation 1: Linear Growth

A significant number of countries in our empirical analysis depicted linear growth patterns. At the same time, such patterns play basically no role in economic theories. If theories are to represent reality, it makes sense to develop such theories. The general pattern is simple and has already been developed in Section 3:

$$y_t = b_0 + (1 + r)t \tag{14}$$

The central theoretical task is to explain the size of $r$ and to develop models that combine the linear growth pattern with such theoretical explanations. Such theories can be developed based on the theories discussed in Section 2. These new models may use the theoretical arguments from established schools of thought — but setting them in a framework of linear rather than exponential growth. For example, technological change could grow linearly rather than exponentially. Or investments could be determined not as a fraction of production but as an absolute value, leading to linear growth. Similar arguments can be made for other determinants of growth, so it should be feasible to develop models of linear growth with well-established theoretical explanations. At the same time, it does not entirely convince to solely focus on developing theories and models of linear growth, that could replace the ones on exponential growth. This approach does not suffice because we have seen that the developments in some economies are indeed best depicted by exponential growth and that this number of countries increases if we take out the years following the financial crisis (see Table 4). This is where the explanation of secular stagnation comes in.

### 5.2. Interpretation 2: Secular Stagnation

In recent years, a debate has emerged on whether several mature economies have entered a phase of secular stagnation — that is, a long term period of low or zero economic growth in GDP. Discussions on an end to economic growth have reemerged periodically throughout economic history. All the major classical economists of the 18th and 19th century had a concept of the steady-state that marked an end to economic expansion (Luks, 2001).[17] A first discussion on secular stagnation took place after the Great Depression, in which Hansen (1939) argued that (1) geographical expansion, (2) seizing population growth and the subsequent change in age structure of the population and (3) less capital-intensive technologies were responsible for declining growth rates. Schumpeter (1941) in contrast focused on unfavorable business conditions for entrepreneurs. The debate came up again in the late 1970s and 1980s. Magdoff and Sweezy argued the lack of investments caused low growth rates and while recovery of a growth path was possible in theory, 'nothing like that is visible on the horizon now' (Sweezy, 1982, p. 9).[18]

The debate has reemerged recently, following the global financial crisis, with contributions from various prominent economists. The general perspective is that growth is lower today than in the past decades, due to specific historical circumstances. Explanations for the current low growth rates in mature economies can be categorized in five groups. Many of the following explanations refer to mature economies in general, while some only apply to specific countries, in particular to the USA.

#### 5.2.1. Slower Technological Change

A first line of argument claims that the major cause for the slow growth rates lies in the pattern of technological change. In recent decades, technological change has increased labor productivity at a slower rate than in previous decades. Gordon (2012) argues that capitalism experienced a first industrial revolution 'between 1750 and 1830', a second between '1870 and 1900' and a third that began 'around 1960' and 'reached its climax in the […] late 1990s' (Gordon, 2012, pp. 1–2). Gordon argues that the technologies of the first and second enabled higher increases in labor productivity than the third — and that this is the explanation for recent low growth rates. In a similar vein, Cowen argues that technological changes in the areas of information and communication technologies cannot reverse this trend.[19] This analysis directly reflects the early growth theories with exogenous technological growth (as explained above), where technological change is the prime determinant of economic growth.

#### 5.2.2. Labor and Human Capital

Within recent discussions on secular stagnation there are two arguments concerning the production factor labor. First, the average number of working hours per capita has decreased. While the average working hours per worker have declined in Europe (not so much in Northern America), the participation rate of women has increased (Maddison, 2006). While reproduction work can in principle be organized via the market and thereby contribute to economic growth in the short run, there appears to be few perspectives for increasing labor productivity in the care sector, due to the nature of the work. Therefore, its contribution to long run growth is limited (Kümmel, 2011).

The decline in average working hours is mainly due to demographic change, which implies an increase in the dependency ratio. This is expected to continue for almost all countries investigated in this paper. Based on these findings, Johansson and Guillemette (2012) concluded that 'population ageing, due to the decline in fertility rates and generalized gains in longevity, has a potentially negative effect on trend growth as it leads to a declining share of the working age population as currently defined (15–64 years)' (Johansson and Guillemette, 2012, p. 13).

Second, it is argued that the benefits of education to increase the

[17] The steady state signifies something different in classical than in neoclassical theories. While it means a constant level of production in the former, it means constant growth of all variables in the latter.

[18] See also Magdoff and Sweezy (1987).

[19] Note that other authors such as Brynjolfsson and McAfee (2014), Ganschar et al. (2013), and Pratt (2015) argue the opposite, that the main effects of the application of information and communication technologies are yet to come and mature economies will experience strong increases in labor productivity in the coming years.

productivity of workers have declined over the past decades and that this trend is likely to continue. Gordon (2014) states that the major reason for this lies in institutions. He argues that in the USA, future 'increases in high school completion rates are prevented by dropping out, especially of minority students' (p. 51) and that the inability of many to finance academic studies is a major problem in improving education. Similarly, Eichengreen (2014) argues that the government has spent too little on public education over the past decades in the USA.

#### 5.2.3. Insufficient Investments

Investments play a crucial role for capital deepening, the application of new technologies and economic growth (compare the Kaleckian and Keynesian growth theories covered above). Eichengreen (2014) argues that low investments by the government, in particular in infrastructure, have been a major reason for overall low investments in the USA. According to Krugman (2014) the decrease in population growth and the subsequent change in the age structure of the population decreases overall investments. The reason is that working aged people buy relatively more capital-intensive goods than senior citizens do and that the demographic change has therefore decreased investments.

#### 5.2.4. Non-competitive Market Structures

Another explanation for decreasing investment rates refers to market structures. Foster (2014) argues — based on earlier work by Baran and Sweezy (1966), Steindl (1976) and Magdoff and Sweezy (1987) — that many mature economies are marked by an increasing concentration of market power and therefore stimulate the formation of oligopolistic and monopolistic market structures. These structures increase profit rates within the concentrated sectors and thereby lower aggregate demand (assuming higher savings out of profits than out of wages). At the same time, investments in these sectors are low as monopolists maximize profits by setting investments and production lower than in competitive markets. Due to low demand, other profitable investment opportunities are scarce, leading to an overall reduced level of investments and an equilibrium below potential output. Foster comes to the conclusion that the outcome of concentrated markets is 'a chronic condition of secular stagnation' (Foster, 2014, p. 87).

#### 5.2.5. Consumption Demand

Consumption and therefore aggregate demand slackens for two reasons: both reasons are based on decreasing marginal consumption expenditures with increasing income. First, the slower but still rising tide does actually not lift all boats equally. 'Rising inequality raises the share of income going to those with a lower propensity to spend' (Summers, 2014a, p. 33) and therefore leads to only minor increases in consumption (see also Gordon, 2014; Eggertsson and Mehrotra, 2014). Second, Reuter (2000) argues that with more and more absolute needs being satisfied, a certain increase in income leads to less increases in consumption throughout different income levels.

These different explanations of secular stagnation hence refer to the different theories of economic growth covered in Section 1. The aspects of technological change, labor and human capital refer to neoclassical theories. The explanation for secular stagnation is that these three aspects grow slower than in the past — and therefore economic growth is lower. The aspects of investments and consumption most directly refer to Keynesian arguments, seeing some form of decrease in demand as major cause of low growth rates. The explanation of market structures refers to Neo-Marxian theories, where the structural change of the economy is the explanation for low growth.

### 5.3. Interpretation 3: A Specific Historical Phase in Economic Development

A similar reaction to the empirical findings of this paper can be based on combining it with the work of economic historians. Maddison (2006) argues that there were five phases of economic growth in Western European countries (Austria, Belgium, Denmark, Finland, France, Germany, Italy, Netherlands, Norway, Sweden, Switzerland and United Kingdom) and Western Offshoots (Australia, New Zealand, Canada and the United States) with different levels of growth rates: 1820–1870, 1870–1913, 1913–1950, 1950–1973 and 1973–1998. (1) Following the industrial revolution, growth of GDP per capita rose well above the low growth rates of the centuries before, at 1.00%/1.42% (for West European countries/Western Offshoots). (2) In the second phase, industrial production was increasingly accompanied by a globalization of markets. That further increased the growth rates to 1.33%/1.81%. (3) The third period was strongly influenced by the two world wars. During this time European countries grew less (0.83%) than the Western Offshoots (1.55%, in particular the USA with 1.61%). (4) The post-war period was characterized by rebuilding and the catching-up of European countries (3.93%) compared to the USA (2.44%). (5) The last period was marked by lower growth rates (1.75%/1.94%) because the effects of rebuilding and catching-up were no longer felt and due to several other effects which are discussed in the next section. The important point is, that the speed of economic growth differed between different historic phases.

Combining such a perspective with the findings above, two different interpretations are plausible. First, the phase investigated (1960–2013) can be classified as one phase that is rather represented by linear growth. In other words: while earlier phases might have had high exponential growth, this phase might depict linear growth. The second possible interpretation is that the phase investigated in fact encompasses two phases with different speeds of growth — and that the finding of linear growth is solely a statistical result of investigating them combined. This interpretation is supported by the result that excluding the years 2008–2013 increases the number of countries depicting exponential growth.

### 5.4. Plural Theories of Economic Growth for Diverse Patterns of Growth

The empirical investigation has shown that it does not suffice to develop theories of exponential growth only. Some countries have depicted exponential growth, and in others the development is better explained by linear growth. Additionally, setting the phase investigated into context with longer historical developments suggests that the phase at hand does not necessarily represent capitalistic economic development in general. If the pattern of economic growth is different between countries and between historic phases, theories of economic growth need to account for this heterogeneity.

As a result, two tasks are central for further work on theories of economic growth. First, a more diverse set of theories and models are needed, so that different models can explain different countries or historic situations. The one-size-fits all solution of prominent theories does not suffice to explain the vast heterogeneity of economic developments over time and space. Second, theoretical explanations are needed to understand why a certain country in a certain time in history depicts a certain pattern of growth — and therefore which model can explain its path of development. Certainly, there is already much research on such questions, for example in literature on economic history and in development economics. This could be utilized to improve theories of economic growth.

## 6. Conclusion

This paper started out with the hypothesis that exponential growth is the perceived *normal* within economics and economic policy discussions. Analysis of the major economic growth theories in Section 2 indicated that this hypothesis holds true. Economic growth theories are built to represent exponential growth. Our empirical analyses in Sections 3 and 4 cast serious doubts on these common views on economic growth: in our main analyses using the time frame of 1960–2013, 11 out of 18 mature economies depict linear rather than exponential growth. Additionally, only 2 of the countries exhibit exponential per-capita growth at the perceived *normal* level of above 1.3%. Therefore, our first conclusion is that *exponential growth does not accurately represent the pattern of mature economies in general.* We observe, however, substantial dependence on the sample period. While the exclusion of the 1960s does not change the results noticeably, the exclusion of the recent crisis (2008–2013) decreases the number of countries whose growth paths rather follow a linear pattern. However, even here the estimated growth rates of the majority of countries (ten out of 18) are below 1.3% and thus apart from commonly discussed growth rates. Further research could improve the investigation by modeling other than linear or exponential trends and by including more data in the future. This suggests that the pattern of growth varies between countries and over time. This insight is supported by longer historical observations (in Section 5). A plural set of theories is needed to explain a diverse set of patterns of growth over time and space. Our results therefore call for future research on economic growth to work on explanations for such different patterns of growth and for why certain patterns take place in certain historic situations.

## Acknowledgments

Funding for this research was granted by the German Federal Ministry of Education and Research within the project "Digitalization and Sustainability" (www.nachhaltige-digitalisierung.de/en/) (Grant # 01UU1607B) and by Deutsche Forschungsgemeinschaft (DFG) within project KO 5269/1-1.

## Appendix A

Table 3
p-Values for unit root tests for the original GDPPC series and its logarithms.

| | ADF-test | ADF-test | KPSS test | KPSS test |
|---|---|---|---|---|
| Country | Log. series | Orig. series | Log. series | Orig. series |
| Australia | 0.2840 | 0.9024 | 0.0251 | 0.0100 |
| Austria | 0.7787 | 0.4320 | 0.0100 | 0.1000 |
| Belgium | 0.9221 | 0.9182 | 0.0100 | 0.0126 |
| Canada | 0.3972 | 0.3698 | 0.0100 | 0.0265 |
| Denmark | 0.8942 | 0.8433 | 0.0100 | 0.0281 |
| Finland | 0.6531 | 0.1995 | 0.0100 | 0.0858 |
| France | 0.6775 | 0.9689 | 0.0100 | 0.0100 |
| Italy | 0.9900 | 0.9900 | 0.0100 | 0.0100 |
| Luxembourg | 0.7471 | 0.5411 | 0.0100 | 0.0100 |
| Netherlands | 0.3313 | 0.5600 | 0.0100 | 0.0135 |
| New Zealand | 0.3428 | 0.4783 | 0.0100 | 0.0100 |
| Norway | 0.9900 | 0.8900 | 0.0100 | 0.0180 |
| Portugal | 0.6544 | 0.6470 | 0.0100 | 0.0200 |
| Spain | 0.9182 | 0.3822 | 0.0100 | 0.0612 |
| Sweden | 0.2203 | 0.5959 | 0.0100 | 0.0100 |
| Switzerland | 0.0237 | 0.0227 | 0.1000 | 0.1000 |
| United Kingdom | 0.5463 | 0.4665 | 0.0452 | 0.0100 |
| United States | 0.6881 | 0.5351 | 0.0100 | 0.0100 |

A linear trend in a log-transformed series corresponds to an exponential growth. The KPSS test in R does not report p-values smaller than 0.01 or bigger than 0.1.

Table 4
Optimal growth rates leading to the optimal ARIMA ($p$,1,$q$) model with drift with respect to different model selection criteria. A growth rate of (close to) zero implies a linear growth model. Time horizon: 1960–2007.

| Country | AIC | AICc | BIC |
|---|---|---|---|
| Australia | 0.0245 | 0.0245 | 0.0245 |
| Austria | 0.0086 | 0.0086 | 0.0086 |
| Belgium | 0.0012 | 0.0012 | 0.0012 |
| Canada | 0.0000 | 0.0000 | 0.0000 |
| Denmark | 0.0037 | 0.0037 | 0.0037 |
| Finland | 0.0282 | 0.0282 | 0.0282 |
| France | 0.0000 | 0.0000 | 0.0000 |
| Italy | 0.0000 | 0.0000 | 0.0000 |
| Luxembourg | 0.0441 | 0.0416 | 0.0416 |
| Netherlands | 0.0159 | 0.0159 | 0.0159 |
| New Zealand | 0.0441 | 0.0441 | 0.0441 |
| Norway | 0.0110 | 0.0110 | 0.0110 |
| Portugal | 0.0000 | 0.0000 | 0.0000 |
| Spain | 0.0012 | 0.0012 | 0.0012 |
| Sweden | 0.0220 | 0.0220 | 0.0220 |
| Switzerland | 0.0331 | 0.0331 | 0.0331 |
| United Kingdom | 0.0318 | 0.0318 | 0.0318 |
| United States | 0.0074 | 0.0074 | 0.0110 |

Table 5
Optimal growth rates leading to the optimal ARIMA ($p$,1,$q$) model with drift with respect to different model selection criteria. A growth rate of (close to) zero implies a linear growth model. Time horizon: 1970–2013.

| Country | AIC | AICc | BIC |
|---|---|---|---|
| Australia | 0.0172 | 0.0172 | 0.0172 |
| Austria | 0.0000 | 0.0000 | 0.0000 |
| Belgium | 0.0000 | 0.0000 | 0.0000 |
| Canada | 0.0000 | 0.0000 | 0.0000 |
| Denmark | 0.0000 | 0.0000 | 0.0000 |
| Finland | 0.0000 | 0.0000 | 0.0000 |
| France | 0.0000 | 0.0000 | 0.0000 |
| Italy | 0.0551 | 0.0000 | 0.0000 |
| Luxembourg | 0.0000 | 0.0000 | 0.0000 |
| Netherlands | 0.0000 | 0.0000 | 0.0000 |
| New Zealand | 0.0135 | 0.0135 | 0.0135 |
| Norway | 0.0000 | 0.0000 | 0.0000 |
| Portugal | 0.0000 | 0.0000 | 0.0000 |
| Spain | 0.0000 | 0.0000 | 0.0000 |
| Sweden | 0.0233 | 0.0135 | 0.0135 |
| Switzerland | 0.0012 | 0.0012 | 0.0012 |
| United Kingdom | 0.0000 | 0.0000 | 0.0000 |
| United States | 0.0000 | 0.0000 | 0.0000 |

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