# Liquidity, Unemployment, and the Stock Market

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# Introduction

# Objective

- We study the role of liquidity constraints and expenditure risk on the relationship between interest-rate spreads, unemployment, and stock prices
- Emerging recognition on the importance of time-varying interest rates and liquidity on job creation

# Motivation: Asset prices and unemployment

1. Stable empirical relationship between unemployment and the stock market (Farmer 2012, Hall 2017) Forecasting unemployment using bivariate VECM
2. Empirical evidence that liquidity constraints matter for unemployment (Mian and Sufi 2010, 2012)
3. Estimates of marginal propensity to consume out of stock market wealth Details

# Motivation: Expenditure risk and liquidity premia

- Significant household expenditure risk: $22 - 30\%$ of HH experience a major out-of-pocket unexpected medical expense, mean value of $\$2400$ (2015 Federal Reserve Report of Economic Well-being)
- Sizable liquidity premia correlated with debt-to-GDP (Krishnamurthy and Vissing-Jorgensen 2012)

# Framework

- Frictional labor market à la Mortensen-Pissarides
- Limited commitment problem and expenditure risk creates liquidity constraint à la Bewley
- Self-insurance through mutual fund of stocks and publicly issued bonds
- Firm revenue depends on early consumption à la Lagos-Wright (2005) and Rocheteau-Wright (2005)

$\Rightarrow$ induces two major channels between stock market and labor market

# Aggregate demand channel

- Higher stock market valuations relax consumers' liquidity constraints
- Extra sales in the product market and greater firm value boost hiring
- Creation of new firms and jobs enhances stock market capitalization

# Interest rate channel

- Higher liquidity boosts real interest rates (lower liquidity premium)
- Firms' profits are discounted at a higher rate
- Firm value and entry fall
- Stock market capitalization declines

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Note: Logic of interest rate channel resembles Aiyagari (1994) but with stock market capitalization in place of capital stock

# Key exercises

1. Qualitative analysis
2. Quantitative analysis
   - Long run
     - Rise in expenditure risk
     - Testing cointegration
   - Short run
     - Bayesian VAR using sign and zero restrictions
     - Impulse responses to early-consumption demand
     - Perfect storm exercise

# Related literature

- Model nests
  - Mortensen-Pissarides
  - Bewley-Aiyagari: idiosyncratic preference shocks with self-insurance

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  - Diamond and Dybvig (1983)–idiosyncratic liquidity shocks
  - Krusell, Mukoyama, and Sahin (2010): DMP with self-insurance against income shocks
  - Branch, Petrosky-Nadeau, and Rocheteau (2016): housing provides collateral through home equity
  - Berentsen-Menzio-Wright (2011): long-run inflation and unemployment

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- Financial accelerator (Kiyotaki and Moore 1997)
- Empirical estimates of MPC out of stock-market returns
  - Majlesi, Di Maggio, and Kermani (2020)
  - Chodorow-Reich, Nenov, and Simsek (2019)

# Structural VAR evidence

# Bayesian VAR with sign and zero restrictions

- Aggregate demand channel implies negative comovement between stock market prices and spread
- Further restrict contemporaneous effect of unemployment to zero (monthly frequency)

Details on SVAR

| Shock | Stock mkt | Spread | Ind. pr. | Cons. | Unemployment |
|---|---|---|---|---|---|
| Stock market | + | - | + | + | 0 |
| Interest rate spread | + | + | + | + | 0 |

Table 1: Identification assumptions. Restrictions only apply on impact.

# Impulse response to a negative stock price shock

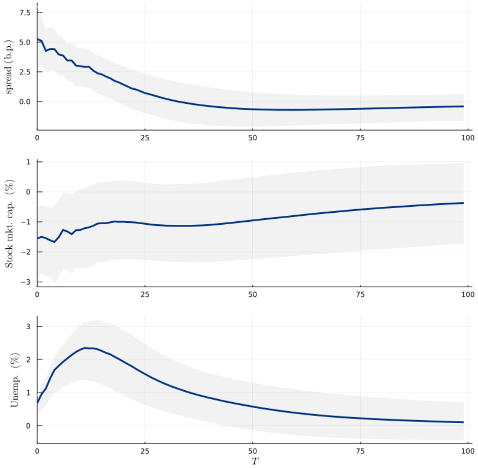


Figure 1: Impulse response to a 1 standard-deviation negative shock to the stock-market valuation. The solid line indicates the point-wise median, and the shaded region represents the $68\%$ probability bands.

Robustness to zero impact response of unemployment

# Environment

# Timing

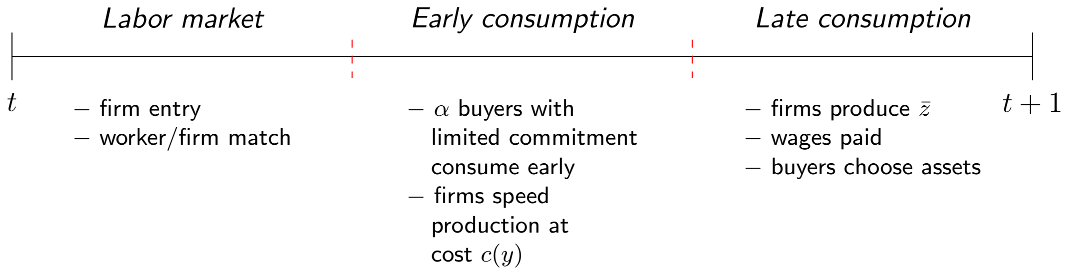


Figure 2: Timing.

# Household

Utility of a HH is

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t[\overbrace{\varepsilon\nu(y_t)}^{\text{preference shock}} + x_t]$$

- $y$: early consumption
- Bernoulli preference shock $\varepsilon : Pr(\varepsilon = 1) = \alpha$
- $x$ numeraire (late) consumption by buyers and workers
- Discount rate $\rho \equiv 1/(1-\beta)$
- $\rho$ is also the rate of return of a risk-free, illiquid bond from one LC period to the next

# Assets

- Fixed supply of one-period real government bonds $A^g$
- Perfectly competitive mutual fund which buys stocks and public bonds and issues risk-free shares
- $r_t$ is the rate of return to such claims from last stage of $t-1$ to last stage of $t$
- A fraction $\lambda$ of HH has access to perfect credit

# Labor market matching and wages

- Matches destroyed at rate $\delta$
- Matching function $m(u, v)$: CRS
- Tightness $\theta = v/u$
- Job finding probability $e = m(1, \theta)$
- Wage $w_1$ determined by Nash bargaining
- Unemployment benefits $w_0$

# Early consumption market

- Firms can produce $y^s$ units for early consumption market at cost $c(y^s)$
- Price $p$ is determined competitively

# Equilibrium

# Late consumption

- Wealth $\omega$ consists of shares of mutual funds net of debt obligations and tax liabilities

$$W(\omega) = \max_{x,a'} \{x + \overbrace{\beta V(a')}^{\text{lifetime utility at stage 2}} \}$$

s.t. $a' = (1 + r')(\omega - x) \geq 0$

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s.t. $a' = (1 + r')(\omega - x) \geq 0$

- Using constraint to eliminate $x$:

$$W(\omega) = \omega + \max_{a' \geq 0} \left\{ -\frac{a'}{1 + r'} + \beta V(a') \right\}$$

- Euler equation

$$\overbrace{1}^{\text{opportunity cost}} = \overbrace{(1 + r)\beta V'(a)}^{\text{discounted marginal utility}}$$

# Early consumption

- Linearity implies

$$V(a) = \alpha\Big[\overbrace{(1-\lambda)\max_{py\leq a}\{v(y)-py\}}^{\text{No credit access}} + \overbrace{\lambda\max_{y\geq 0}\{v(y)-py\}}^{\text{Credit access}}\Big] + a - \tau + W(0).$$

  for taxes $\tau$

- Let $y^*$ be optimal consumption under perfect credit and $\hat{y}$ without perfect credit

$$y^* = v'^{-1}(p)$$
$$\hat{y} = \min\{y^*, a/p\}$$

# Revenue, market clearing, and output

- Expected revenue of a firm

$$z = \bar{z} + \overbrace{\max_y \{py - c(y)\}}^{\text{endogenous productivity}}$$

- Perfect competition implies $p = c'(y^s) \Leftrightarrow y^s = c'^{-1}(p)$
- Market clearing: $\overbrace{ny^s}^{\text{supply}} = \overbrace{\alpha[\lambda y^* + (1-\lambda)\hat{y}]}^{\text{demand}}$
- GDP $= nz$

Why perfect competition?

# Interest rate and liquidity premium

- Evaluate $V'(a)$ and plug into Euler equation

$$\overbrace{\frac{\rho - r}{1+r}}^{\text{marginal cost of asset}} = \overbrace{\alpha(1-\lambda)\left[\frac{v'(\hat{y})}{c'(y^s)} - 1\right]}^{\text{expected marginal benefit}}$$

Derivation

- $\rho - r$: liquidity premium

# Value of firm

- Value of a firm

$$J = \overbrace{z}^{\text{agg. dem. channel}} - w_1 + (1-\delta) \overbrace{\frac{J'}{1+r'}}^{\text{interest rate channel}}$$

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$$J = \overbrace{z}^{\text{agg. dem. channel}} - w_1 + (1-\delta)\overbrace{\frac{J'}{1+r'}}^{\text{interest rate channel}}$$

- Infinite series representation

$$J_t = \sum_{i=0}^{\infty} \frac{(1-\delta)^i (z_{t+i} - w_1)}{R_{t+i}}$$

given sequence of gross real interest rates

$$R_t = 1$$
$$R_{t+i} = (1+r_{t+1})(1+r_{t+2})\cdots(1+r_{t+i}) \quad \text{for } i \geq 1$$

# Labor market

- Rate of return of investing in new firm: $q_{t+1}J_{t+1}/k$
- Rate of return of purchasing a share of a mutual fund: $1 + r_{t+1}$
- Free entry (no-arbitrage) condition

$$(1 + r_{t+1})k = q_{t+1}J_{t+1}$$

Implied Job Creation Condition

# Equilibrium wage

- Wage is determined via generalized Nash bargaining between the worker and the firm
- Worker has bargaining power $\phi$

$$w_{1t} = (1-\phi)\, w_0 + \phi\left[z_t + \beta\,(1+r_{t+1}) k\theta_t\right] + \frac{\phi\,(1-\delta)\,k}{q\,(\theta_t)}\left[1-\beta\,(1+r_{t+1})\right]$$

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- Firms discount at rate $1/(1+r)$ rather than $\beta$
- If the liquidity constraint does not bind, then $\beta(1+r_{t+1})=1$, and wage coincides with Mortensen-Pissarides:

$$w_1 = (1-\phi)w_0 + \phi(z+k\theta)$$

Derivation

# Market capitalization

- Link stock market, unemployment, and interest rate

$$\overbrace{a}^{\text{Portfolio value}} = nJ + A^g = \frac{n(1+r)k}{q(\theta)} + A^g$$

- Positive relationship between stock market capitalization, employment, and interest rates

# Equilibrium

We define an equilibrium as a bounded sequence, $\{J_t, \theta_t, n_t, p_t, r_t, w_t\}_{t=0}^{+\infty}$, that solves:

Firm Bellman $\quad J_t = \dfrac{(1+r_t)k}{q(\theta_t)} = \bar{z} + \max_y \{p_t y - c(y)\} - w_1 + (1-\delta)\dfrac{J_{t+1}}{1+r_{t+1}}$

Pricing $\quad c'^{-1}(p_t) = \dfrac{\alpha}{n_t}\left[\lambda v'^{-1}(p_t) + (1-\lambda)\min\left\{v'^{-1}(p_t), \dfrac{n_t J_t + A^g}{p_t}\right\}\right]$

Interest rate $\quad \dfrac{\rho - r_t}{1+r_t} = \alpha(1-\lambda)\left[\dfrac{v'\left(\frac{n_t J_t + A^g}{p_t}\right)}{p_t} - 1\right]^+$

Employment LOM $\quad n_{t+1} = (1-\delta)n_t + m(1, \theta_{t+1})(1-n_t),$

Wage $\quad w_{1t} = (1-\phi)\, w_0 + \phi\left[z_t + \beta\,(1+r_{t+1})k\theta_t\right] + \dfrac{\phi\,(1-\delta)\,k}{q\,(\theta_t)}\left[1 - \beta\,(1+r_{t+1})\right]$

# Qualitative analysis: deconstructing the model

# Deconstructing the model

- Start with Mortensen-Pissarides model and add one ingredient at a time
- Simplify by assuming a fixed wage temporarily
- Use continuous time to represent dynamics graphically via phase diagrams

# Equilibrium in continuous time

Equilibrium is a bounded sequence $(J, r, p, n)$ such that, given $n_0$, satisfies

$$(r+\delta)\,J = \bar{z} + \max_{y}\left\{py - c(y)\right\} - w_1 + \dot{J}$$

$$c'^{-1}(p) = \frac{\alpha}{n}\left[\lambda v'^{-1}(p) + (1-\lambda)\min\left\{v'^{-1}(p), \frac{nJ + A^g}{p}\right\}\right]$$

$$\rho - r = \alpha(1-\lambda)\left[\frac{v'\left(\frac{nJ+A^g}{p}\right)}{p} - 1\right]^+$$

$$\dot{n} = m\left[1, \theta(J)\right](1-n) - \delta n.$$

where $\theta(J)$ solves $J = k/q(\theta)$

# Special case: $\alpha = 0$ (MP)

- Stocks and bonds provide no liquidity: $r = \rho, z = \bar{z}$
- Equilibrium can be reduced to a pair $(J, n)$ that solves

$$(\rho + \delta)J = \bar{z} - w_1 + \dot{J}$$
$$\dot{n} = m\left[1, \theta(J)\right](1 - n) - \delta n,$$

  where $\theta(J)$ solves $J = k/q(\theta)$
- J-isocline is horizontal (independent of employment)
- n-isocline slopes upward since higher market value of firms induces greater market tightness

# M-P with early consumption and perfect credit

- Preference shocks $(\alpha > 0)$ but perfect credit in early consumption stage
- Equilibrium is a list $\{J, p, y^s, n\}$ such that

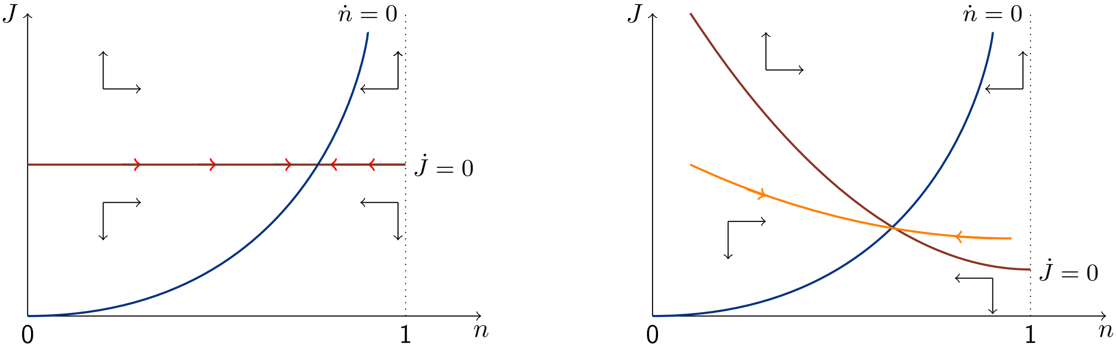
$$(\rho + \delta)\, J = \bar{z} + \max_{y} \{py - c(y)\} - w_1 + \dot{J}$$

$$v'\left(\frac{ny^s}{\alpha}\right) = p = c'\left(y^s\right)$$

$$\dot{n} = m\left[1, \theta(J)\right](1 - n) - \delta n$$

- $p$ is decreasing in $n$: J-isocline is decreasing in $n$

# Phase diagrams of one-good and two-good M-P models




(a) MP model.

(b) 2-good MP model with convex cost.

Figure 3: Phase diagrams: MP models.

# Bewley-Aiyagari (MP with limited commitment)

- $c(y) = y \Rightarrow p = 1, \lambda = 0$: one-good economy, shut down aggregate demand channel
- Equilibrium is a list $(J, r, n)$ such that

$$\begin{aligned}(r+\delta)\,J &= \bar{z} - w_1 + \dot{J} \\ \rho - r &= \alpha\left[v'\left(nJ + A^g\right) - 1\right]^+ \\ \dot{n} &= m\left[1, \theta(J)\right](1-n) - \delta n.\end{aligned}$$

- $r \in (-\delta, \rho)$

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- $r \in (-\delta, \rho)$
- Remark. Real interest rate, $r$, is endogenous through liquidity, but firm revenue is exogenous.
- J-isocline satisfies $[r(nJ + A^g) + \delta]J = \bar{z} - w_1$ and depends negatively on $n$
- $\uparrow n$ implies higher market capitalization for given $J$, reduces liquidity constraints and raises $r$, which lowers $J$

# Phase Diagram: Bewley-Aiyagari

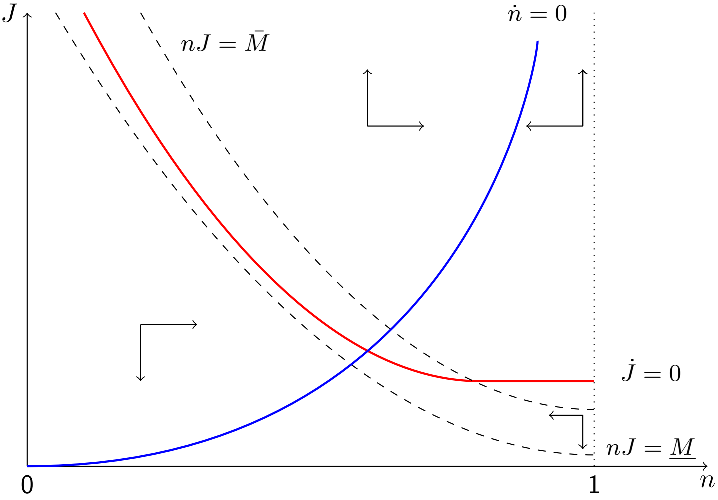


Figure 4: Phase diagram: Bewley-Aiyagari where $c(y) = y$.

# Nullclines for full model

Simplify system to two equations using functions $z(\overset{-}{n}, \overset{+}{J}, \overset{+}{A^g})$ and $r(\overset{+}{n}, \overset{+}{J}, \overset{+}{A^g})$

$$\dot{J} = \left[ r(\overset{+}{n}, \overset{+}{J}, \overset{+}{A^g}) + \delta \right] J + w_1 - z(\overset{-}{n}, \overset{+}{J}, \overset{+}{A^g}) \equiv f(n, J, A^g)$$

$$\dot{n} = m\left[1, \theta(\overset{+}{J})\right](1-n) - \delta n \equiv g(n, J).$$

Derivation of functions

# J-nullcline

$$J = \frac{z(\overset{-}{n}, \overset{+}{J}, \overset{+}{A^g}) - w_1}{r(\overset{+}{n}, \overset{+}{J}, \overset{+}{A^g}) + \delta}$$

- Expression is monotone declining in $n$ but can be non-monotone in $J$
- Increase in government bonds $A^g$ raises both interest rates and revenue–ambiguous effect on $J$

Comparison to a pure monetary economy | Multiplicity in the literature

# Quantitative analysis

 Quantitative analysis

# Functional forms

$$m(s,o) = As^{\xi}o^{1-\xi}$$
$$v(y) = B\log y$$
$$c(y) = C\frac{y^{1+\sigma}}{1+\sigma}$$

- Can show that $C$ drops out of equilibrium conditions: set $C = 1$ WLOG

# Simplification of equilibrium conditons

Under these functional forms, Firm Bellman and liquidity premium simplify to

$$J_t = \overline{z} + \frac{\sigma}{1+\sigma}\frac{\alpha}{n_t}\left[\lambda B + (1-\lambda)\min\{B, n_t J_t + A^g\}\right] - w_{1t} + \frac{(1-\delta)J_{t+1}}{1+r_{t+1}}$$

$$\frac{\rho - r_t}{1+r_t} = \alpha(1-\lambda)\left[\frac{B}{n_t J_t + A^g} - 1\right]^+$$

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$$\frac{\rho - r_t}{1+r_t} = \alpha(1-\lambda)\left[\frac{B}{n_t J_t + A^g} - 1\right]^+$$

Note that

- $\alpha = 0$ implies $z = \overline{z}, r = \rho$
- $\sigma = 0$ implies $z = \overline{z}$

# Calibration

| Parameter | Interpretation | Values | Calibration Strategy |
|---|---|---|---|
| $\delta$ | Separation rate | 0.03045 | Mean separation rate |
| $\overline{z}$ | Exogenous productivity | 1 | Normalization |
| $w_0$ | Unemployment insurance | 0.7186 | Replacement ratio |
| $\phi$ | Bargaining power | 0.04702 | Treasury demand |
| $\rho$ | discount rate | 0.002466 | Risk free rate |
| $\alpha$ | prob. preference shock | 0.004449 | Treasury demand |
| $B$ | EC utility level parameter | 16.65 | Treasury demand |
| $\sigma$ | elasticity of marginal cost | 0.2 | Price to average cost |
| $k$ | Vacancy posting cost | 4.413 | Consistency with market tightness |
| $A$ | Matching function level parameter | 0.5631 | Job finding rate |
| $\xi$ | Matching function elasticity $u$ | 0.5 | Estimates in Petrongolo and Pissarides (2001) |
| $A^g$ | Public debt | 4.529 | Treasury demand |
| $\lambda$ | Prob. perfect credit | 0.8 | Fraction of HH with credit access to replace income |

Table 2: Parameterization. Monthly frequency.

Data sources for motivation and calibration Details on calibration

# The Treasury demand curve

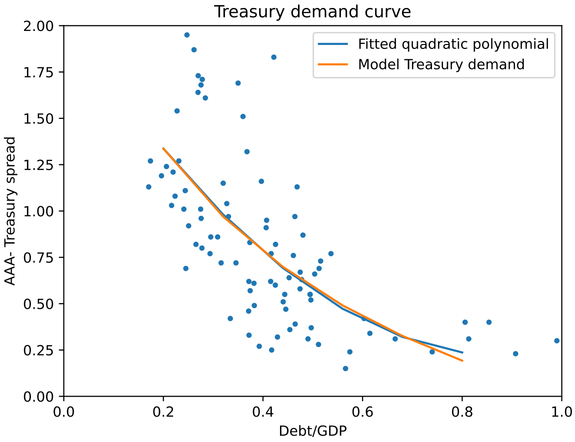


Figure 5: Treasury demand curve. The original data is taken from Krishnamurthy and Vissing-Jorgensen (2012), and coincides with Figure 1 in the article. We fit a quadratic polynomial by least squares, and overlay the Treasury demand curve from the model. Debt-to-GDP is expressed at an annual frequency, and the interest rate spread is expressed in annual percentage units.

# Key steady-state features

- Effective liquidity $nJ + A^g \approx 9.6 < 1.7 \approx B$
- Interest rate spread $\approx 77bp$
- Probability of monthly preference shock $\approx 0.445\%$ or $5.2\%$ annually
- Labor share $= w_1/z \approx 82\%$
- Share private to total liquidity $nJ/(nJ + Ag) \approx 53\%$
- $\epsilon_{n,M} = \frac{\partial \log n}{\partial M} \approx 2.38$
  $\Rightarrow$ job creation is elastic with respect to private liquidity

# Steady state at calibrated parameter values

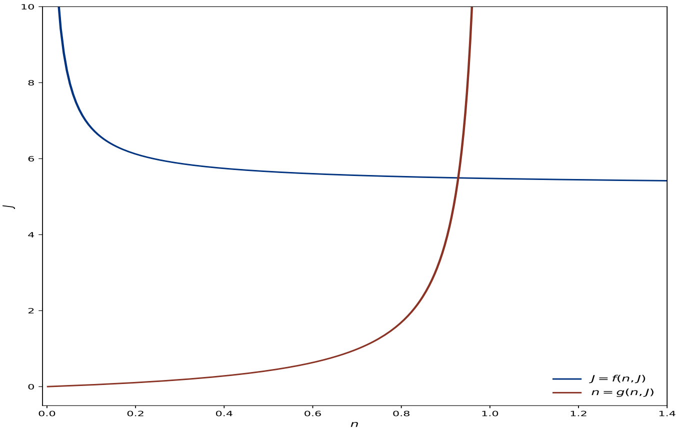


Figure 6: Steady-state at calibrated parameter values

# Decomposing the interest-rate and aggregate-demand channels

- Consider rise in idiosyncratic risk ($\alpha$)
- Bewley-Aiyagari: real interest rate falls and boosts firm values
- Full model: rise in $\alpha$ also increases relative price $p$, revenue $z$, entry $v$, and the real interest rate $r$

# Higher idiosyncratic risk: decomposing the channels

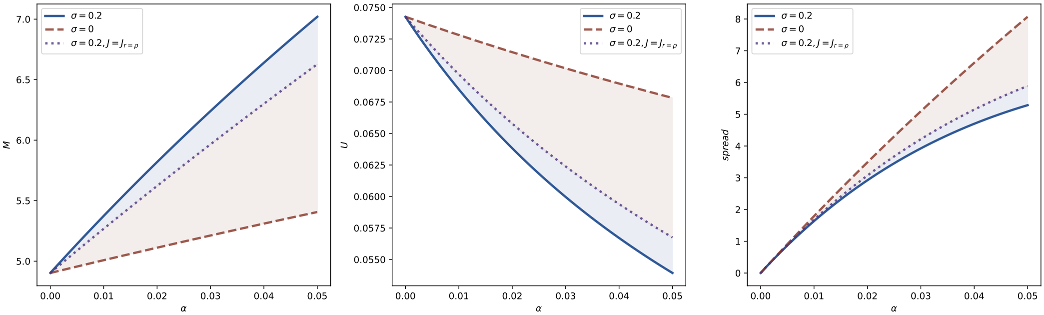


Figure 7: Steady-state values of full model, Bewley-Aiyagari($\sigma = 0$), and model with firms' profits discounted at rate $\rho$. The variables include the stock market capitalization $M$, unemployment $U$, and interest rate spread for different values of $\alpha$. The liquidity premium been annualized and expressed in interest rate form by multiplying by $12 \times 100$.

# Cointegration

- Farmer (2012) provides evidence that the stock market and unemployment are cointegrated from 1959-1979: non-stationary but a linear combination of each other
- Include interest rate spread and evaluate over 1959-2019
- Hypothesized cointegration relationship:

$$\beta_1 u_t + \beta_2 M_t + \beta_3 s_t = 0$$

# Cointegration

| Coint. coeff.'s | $\beta_1$ | $\beta_2$ | $\beta_3$ | $J(0)$ |
|---|---|---|---|---|
| Unemp. & Stock only | $1.0^*$ | 0.55 | $0.0^*$ | 3.498 |
| | | (0.138) | | |
| Unemp., Stock, and Spread | $1.0^*$ | 0.60 | 1.4849 | $16.31^{**}$ |
| | | (0.397) | (0.389) | |

Table 3: Long-run relationship between unemployment, stock market capitalization, and liquidity premia (1959-2019): $\beta_1 u_t + \beta_2 M_t + \beta_3 s_t = 0$. $J(0)$ refers to the Johansen max. eigenvalue test for the null of no cointegrating relationship. $*$ denotes normalization, $**$ is significance at 10% level. $u_t$ is the logit transformation of UNRATE, following Farmer(2012).

- Null hypothesis: no cointegration
- Reject the null at the $10\%$ level for the three variables (but not for two)
- Unemployment is negatively related to the stock market cap and interest-rate spread in the long run

# Modeling expectations

- We study effect of one-time shocks
- Rational expectations requires agents to understand dynamic evolution and behavior of all agents
- Adaptive learning (Marcet and Sargent 1989, Evans and Honkapohja 2001) replaces rational expectations with cognitive consistency principle
  - agents should be modeled as a good economist who formulates and adapts a well-specified forecast rule about aggregates

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    - agents should be modeled as a good economist who formulates and adapts a well-specified forecast rule about aggregates
- Why least-squares learning?
    - helps select among multiple equilibria
    - generates persistence

# Law of motion under adaptive expectations

- Temporary equilibrium (taking expectations as given)

$$J_t = z_t - w_t + \frac{(1-\delta)M_{t+1}^e}{n_{t+1}^e\left(1+r_{t+1}^e\right)}$$

$$n_t = (1-\delta)n_{t-1} + A^{1/\xi}\left[\frac{J_t}{k\left(1+r_t\right)}\right]^{(1-\xi)/\xi}(1-n_{t-1})$$

letting $r_{t+1}^e = r(M_{t+1}^e, n_{t+1}^e)$

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letting $r^e_{t+1} = r(M^e_{t+1}, n^e_{t+1})$

- Steady-state learning dictates that these forecasts should come from a geometrically weighted average of past data

$$M^e_{t+1} = M^e_t + \gamma(M_{t-1} - M^e_t)$$
$$n^e_{t+1} = n^e_t + \gamma(n_{t-1} - n^e_t)$$

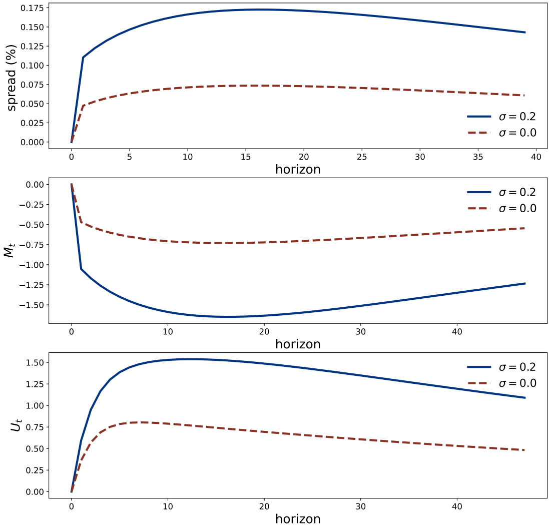
where $\gamma$ is the constant-gain coefficient

- Timing
  - Expectations are formed given most recent data point $(M_{t-1}, n_{t-1})$
  - Agents decide and markets clear: determine $t$-period variable

# Parameters for dynamics

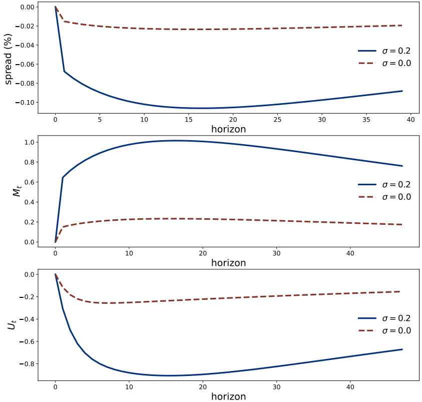
- Let $\mu < 1$ be the rate of decay of shock
- Set $\gamma = 0.2$ and $\mu = 0.9$ (closest match to stock-price SVAR)
- Roughly, $\gamma = 0.2$ implies agents weight most heavily the last six months of data

# Demand shocks: a negative shock to $B$



# Expenditure shock: positive shock to $\alpha$

- Consider $20\%$ innovation to $\alpha$
- temporary increase of about $1\%$ of the population exposed to an expenditure shock over $12$ months



# Counterfactual: a perfect storm

- Unique steady state in baseline calibration
- Multiple steady states can arise under
  - Low (exogenous) productivity $\overline{z}$ relative to wage
  - Low availability of public liquidity $A^g$
  - Strong intensive/extensive margins to aggregate demand

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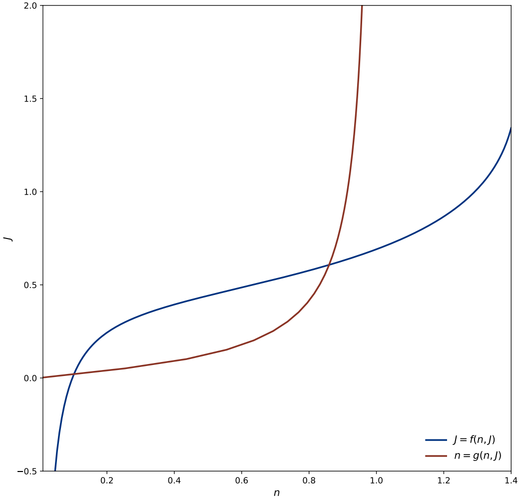
  - Low availability of public liquidity $A^g$
  - Strong intensive/extensive margins to aggregate demand
- Intuitively, firms depend strongly on early-consumption demand, and their value strongly impacts HH liquidity
- Call this situation a 'perfect storm'
- Set $\alpha = 0.04, B = 18.0, \overline{z} = 0.8, A^g = 1.25$
- Fraction of consumers facing expenditure risk jumps from $0.4\%$ to $4\%$, there is a $20\%$ reduction in aggregate productivity, and public liquidity drops by $25\%$

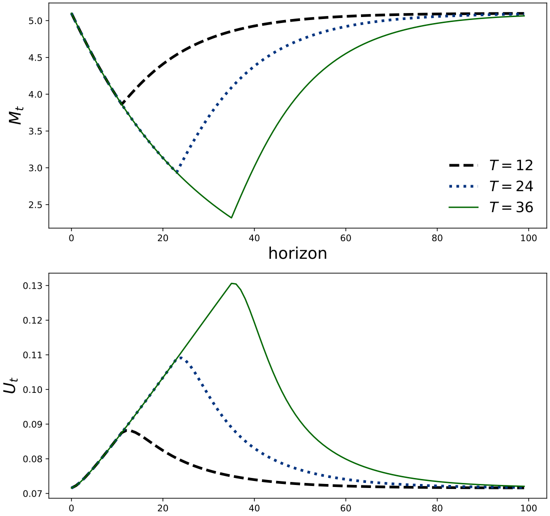
# Steady states under perfect storm



# Perfect storm

- Generates a crash steady state
- Higher steady state is determinate and lower steady state is indeterminate
- Only high steady state is stable under learning
- Assume parameters only last for finite period of time $(12, 24, 36)$ months and then revert to normal

# Perfect storm



# Conclusion

- We examine how expenditure risk and liquidity constraints impact the comovement of the labor market and stock market
- Stock-market valuations relax HH liquidity constraints and induce firms to create more jobs
- Examined effects of higher idiosyncratic risk and decompose aggregate-demand and interest-rate channels
- Identified (negative) stock-market shock raises interest-rate spread and unemployment
- Provided some evidence of cointegration of unemployment, the stock market cap, and the interest rate spread
- A perfect storm of shocks induces multiplicity and affects transition dynamics

# Extensions

- In practice, much equity is held in retirement accounts and is illiquid
- Incorporate money, make stock partially illiquid, and study Phillips curve
- Study effects of precautionary savings from labor income risk with heterogeneous agents $\Rightarrow$ Important for cross-sectional and aggregate implications of credit crunch as Guerrieri and Lorenzoni (2017)

Thank you!

# Appendix

# Forecasting unemployment using bivariate VECM

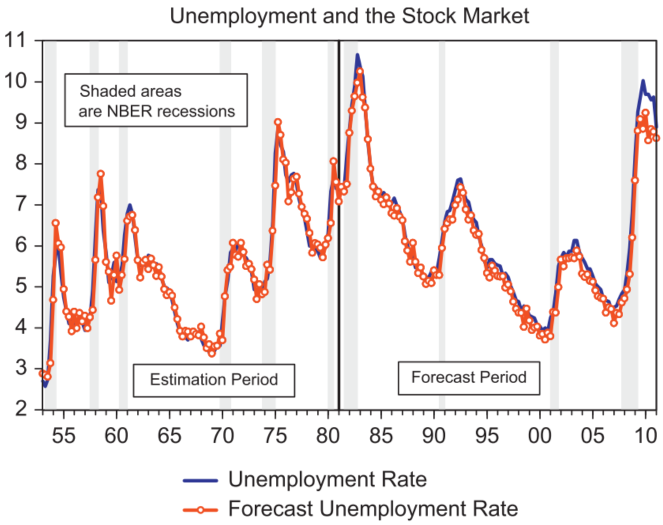



Figure 12: Figure 6 from Farmer (2012).

Back to motivation

# More details on liquidity premia of T-bills

- Independent variable is logarithm of debt-to-GDP ratio
- Dependent variable is bond yield spread measured percentage-wise
- Spread is between long-term AAA- corporate bonds and long-term Treasuries
- Estimate of $-0.746$ by Krishnamurthy and Vissing-Jorgensen (2012) implies that a $1\%$ increase in debt-to-GDP reduces the liquidity premium by $-0.746\%/100$, or $-0.746$ basis points
- Gorton (2012) shows that government debt is strongly negatively correlated with privately produced assets

Back to motivation

# Implementation

- Bayesian algorithm developed by Arias, Rubio-Ramirez, and Waggoner (2018)

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- Key features
  - sign restriction via QR decomposition in Rubio-Ramirez, Waggoner, and Zha (2010)
  - Draws from conjugate uniform-normal-inverse-Wishart posterior over reduced form parameterization
  - Transforms draws into structural parameterization

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  - Draws from conjugate uniform-normal-inverse-Wishart posterior over reduced form parameterization
  - Transforms draws into structural parameterization
- Does not require Metropolis-Hastings algorithm
- Does not impose penalty function as in Mountford and Uhlig (2009)
- Enter variables in log levels (other than spread and unemployment rate)

# Details on SVAR

- Normalize consumption by population and consumer price index
- Use longer horizon of stock price data from Robert Shiller's webpage and normalize by nominal wage

Back to main slides

# Stock market capitalization, consumption, and the labor market

- Majlesi, Di Maggio, and Kermani (2020) analyze Swedish administrative data and find a MPC out of unrealized capital gains of $23\%$ for the bottom half of the wealth distribution and $3\%$ for the top half
- Chodorow-Reich, Nenov, and Simsek (2019) find that a $20\%$ increase in stock market valuations increase aggregate hours by $0.7\%$ and the aggregate labor bill by $1.7\%$

Back to empirical motivation

# Liquidity constraints and unemployment (Mian and Sufi 2012)

- A one standard-deviation increase in the 2006 debt-to-income ratio of a county is associated with a 3 pp drop in non-tradable employment from 2007-2009
- Decline in aggregate demand driven by household balance sheet shocks accounts for $65\%$ of jobs lost (4 million)
- Rule out an important role for uncertainty shocks or structural shocks

Back to empirical motivation

# Robustness to zero-impact response of unemployment

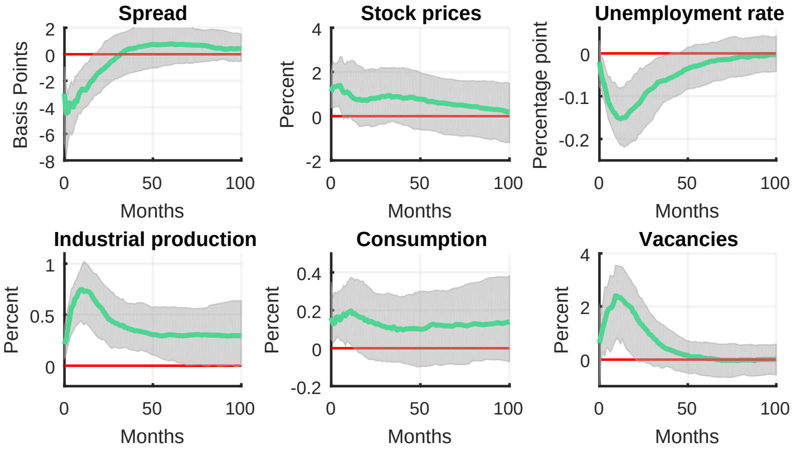


Figure 13: Impulse response to a positive unit standard-deviation shock to the stock-market valuation.

# Derivation of interest rate equation

- Differentiate DM value function

$$V'(a) = \alpha(1-\lambda)(v'(y) - p)\frac{dy}{da} + 1$$

- Use fact that $dy/da = p = c'(y^s)$

$$V'(a) = \alpha(1-\lambda)\left(\frac{v'(y)}{c'(y^s)} - 1\right) + 1$$

- Multiply both sides by $\beta = 1/(1+\rho)$ and substitute Euler equation

$$\frac{1}{1+r} = \alpha\beta(1-\lambda)\left(\frac{v'(y)}{c'(y^s)} - 1\right) + \beta$$

- Simplify

$$\frac{\rho - r}{1+r} = \alpha(1-\lambda)\left(\frac{v'(y)}{c'(y^s)} - 1\right)$$

Back to interest rate equation

# Job creation condition

- Firm Bellman and free entry condition can be rearranged as the job creation condition:

$$\overbrace{\frac{k}{q(\theta_t)}}^{\text{Average hiring cost}} = \overbrace{\frac{1}{1+r_t}\left\{z_t - w_1 + (1-\delta)\frac{k}{q(\theta_{t+1})}\right\}}^{\text{Present discounted value of job}}$$

- Tightness $\theta_t$ rises with endogenous $z_t$ and falls with $r_t$
- Job creation depends on endogenous productivity and discounting
- Infinite series representation

$$\frac{k}{q(\theta_t)} = \sum_{i=0}^{\infty}(1-\delta)^i \frac{z_{t+i} - w_1}{(1+r_t)\cdots(1+r_{t+i})}$$

Back to labor market

# Derivation of functions

- Define first-best consumption $y^*(n)$ as solution to $v'(y^*) = c'(\alpha y^*/n)$
- Let $y^{cons}(n, J, A^g)$ be firm supply under constrained HH:

$$y^{cons} = \frac{\alpha}{n}\left[\lambda v'^{-1}(c'(y^{cons})) + (1-\lambda)\frac{nJ + A^g}{c'(y^{cons})}\right]$$

- Optimal firm supply function

$$y^s(n, J, A^g) = \min\{y^{cons}(n, J, A^g), (\alpha/n)y^*(n)\}$$

- Pricing function $p(n, J, A^g) = c'[y^s(n, J, A^g)]$

# Derivation of functions

- Revenue function

$$z(n, J, A^g) = \bar{z} + p(n, J, A^g) y^s(n, J, A^g) - c[y^s(n, J, A^g)].$$

- Interest-rate function

$$r(n, J, A^g) = \rho - \alpha(1 - \lambda) \left[ \frac{v'\left(\frac{nJ + A^g}{p(n, J, A^g)}\right)}{p(n, J, A^g)} - 1 \right]^+ ,$$

Back to nullclines

# Why perfect competition?

- We use perfect competition because it isolates the aggregate demand channel in this model and is simple
- Monopolistic competition: additional aggregate demand externality as households can better diversify their consumption with more firms
- Bargaining: effects of endogenous markup
- Goods market frictions can also be used to endogenize utilization and link consumption demand and firm revenue

Back to market clearing

# Comparison to pure monetary economy

- Comparison to analogue of Berentsen, Menzio, and Wright (2011) with Walrasian price taking, $\lambda = 0$
- Let $\pi$ denote the growth of the money supply

$$(\rho + \delta)J = \overline{z} + \max_{y} \{py - c(y)\} - w_1$$
$$p = c'(y^s)$$
$$\rho + \pi = \alpha \left[ \frac{v'(ny^s/\alpha)}{c'(y^s)} - 1 \right]$$
$$\delta n = m[1, \theta(J)](1 - n)$$

- Real balances effect: inflation reduces output, relative price, revenue, and employment
- Our framework endogenizes the real interest rate and links relative price and revenue to stock market capitalization

Back to nullclines

# Labor search models with multiplicity

- Schaal and Dumouchel (2016): aggregate demand externality from monopolistic competition
- Kaplan and Menzio (2016)
  - Unemployed workers spend more time searching for lower goods/can pay lower prices
  - As firms hire workers, employment rises, shopping time falls, and markups rise
  - Induces firm entry
- Silva (2020): procyclicality of debt limits through endogenous borrowing constraints
- Multiplicity in this paper does not require monopolistic competition, differential shopping time, or endogenous borrowing constraints Back to logic of multiplicity

# Data sources

| Variable | Source |
|---|---|
| Stock market capitalization | Wilshire 5000 (in logs), FRED code WILL5000INDFC |
| Buffet measure of market capitalizaton | FRED code NCBEILQ027S |
| Nominal wage | A576RC1 |
| Total unemployed | FRED code UNEMPLOY |
| Total employed | FRED code CE160V |
| Average hourly earnings of all employees | Fred code CES0500000003 |
| Average weekly hours of all employees | Fred code AWHAETP |
| Unemployed less than $5$ weeks | FRED code UNEMPLT5 |
| Vacancies | https://www.briancjenkins.com/dmp-model |
| Moody's AAA | FRED code AAA |
| Long-term government bonds | FRED code LTGOVTBD |
| Treasury bonds with 20-year maturities | FRED code GS20 |

Table 4: Data sources used in motivating evidence and model calibration.

Back to calibration

## Derivation of wage equation

- Let $\tilde{J}_t$ be the corresponding expected present-value of the firm before the labor market opens. For the firms, we have

$$\tilde{J}_t = z_t - w_1 + (1-\delta)\frac{\tilde{J}_{t+1}}{1+r_{t+1}} + \delta\frac{\tilde{J}_{v,t+1}}{1+r_{t+1}}$$
$$\tilde{J}_{vt} = -k + q(\theta_t)\frac{\tilde{J}_{t+1}}{1+r_{t+1}}$$

- The free entry condition implies that $\tilde{J}_{vt} = 0$, or that

$$\tilde{J}_{t+1} = \frac{(1+r_{t+1})k}{q(\theta_t)}$$

- It follows that $\tilde{J}_t = J_t$, as we previously defined it.

# Derivation of wage equation

- The Nash bargaining problem solves

$$w_{1t} = \arg\max(U_{1t}(w_t) - U_{0t})^{\phi}(J_t(w_t) - \tilde{J}_{vt})^{1-\phi}$$

  Using the fact that $w_{1t}$ enters linearly in both $U_1(w_t)$ and $J_t(w_t)$, we have

$$\begin{aligned}(1-\phi)(U_{1t} - U_{0t}) &= \phi J_t \\ \Leftrightarrow U_{1t} - U_{0t} &= \phi S_t\end{aligned}$$

  where $S_t = U_{1t} - U_{0t} + J_t$ is the joint surplus.
- Straightforward calculations lead to the Nash wage expression. Back to labor market