Productive demand, sectoral comovement, and total capacity utilization

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Introduction

Contents

1. Perennial question: what drives business cycles?
2. Debate of technology vs. demand shocks closely tied to role of economic slack/capacity utilization and endogeneity of Solow residual
    - Evans (1992) shows that money, interest rates, and government expenditure Granger cause the Solow residual
    - Basu, Fernald, and Kimball (2006) construct a measure of technological change using structural estimation and finds that it behaves very differently than Solow residual

Questions

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Questions and key contribution

- Bai, Rios-Rull, and Storesletten (2024) use goods market frictions to impart productive role for demand in estimated model
- However, framework relies on shopping time data both for calibration and time series properties, and does not make use of capacity utilization
  - Shocks to goods market frictions can also rise from fluctuations in matching efficiency, which cannot be separately identified
  - Shopping time can be contaminated with leisure
- Key contribution of this paper: use capacity utilization jointly with sectoral data to investigate contribution of demand shocks under goods market frictions

Key findings

We estimate a multisector model with goods market frictions using Bayesian techniques and show

1. Key novel parameters associated with the transmission mechanism are economically significant and well-identified
2. Demand shocks explain a majority of the variation of output, the Solow residual, and utilization
3. Model fits the data reasonably well, including major sectoral variables (sectoral comovement) $\Rightarrow$ search demand shocks uniquely generate three-way comovement of utilization rates and Solow residual

Demand shocks and effect on measured productivity

- In a standard neoclassical model, prices adjust so that all produced output is sold
  $\Rightarrow$ output is just a function of capital and labor
- Under goods market frictions, output depends on arrival rate of customers
- Increases in shopping effort—whether exogenous or in response to other economic shocks—generate more matches and higher capacity utilization
- Raises TFP and output
- Reverses causality between consumption and TFP relative to neoclassical model
- Mechanism reflects Keynes' idea of demand-determined output without relying on nominal rigidity

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Capacity utilization data

- Total capacity utilization from Federal Reserve Board is the ratio of an output index to a capacity index
- Coverage
  - 89 detailed industries (71 manufacturing, 16 mining, 2 utilities)
  - Primarily correspond to industries at the 3 or 4-digit NAICS
  - Estimates are available for various groups (durables and non-durables, total manufacturing, mining, utilities, and total industry)
  - Not available for services
- Source data
  - Capacity data reported in physical units from government sources, trade sources
  - Responses to the Bureau of the Census's Quarterly Survey of Plant Capacity (QSPC)
  - Trends through peaks in production for a few mining and petroleum series
- Compared to Fernald utilization measure, does not require assuming constant returns to scale and zero profits

Motivation: Utilization measures and output

- Utilization measures comove positively and are procyclical
- Utilization in durables is significantly more volatile than non-durables

Second moments (growth rates)

Variable	Symbol	SD(x)	STD(x)/STD(Y)	Cor(x, I)	Cor(x, n_I)	Cor(x,x$_{-1}$)
Real GDP	Y	0.87	1.00	0.94	0.70	0.47
Real Consumption	C	0.44	0.51	0.54	0.44	0.48
Real Investment	I	2.14	2.46	1.00	0.73	0.41
Labor in Consumption	n_c	0.57	0.66	0.66	0.87	0.67
Labor in Investment	n_i	1.94	2.23	0.73	1.00	0.64
Labor productivity	Y/N	0.64	0.73	0.36	-0.28	0.10
Relative price of investment	p_i	0.51	0.58	-0.28	-0.22	0.44
Utilization in Durables	$util_d$	2.27	2.61	0.69	0.84	0.55
Utilization in Non-durables	$util_{nd}$	1.26	1.45	0.61	0.65	0.51

Table 1: Time range: $1964Q1 - 2019Q4$. Each underlying series is expressed in 100 quarterly log deviations. Here output is defined as the sum of consumption and investment.

Utilization measures and output

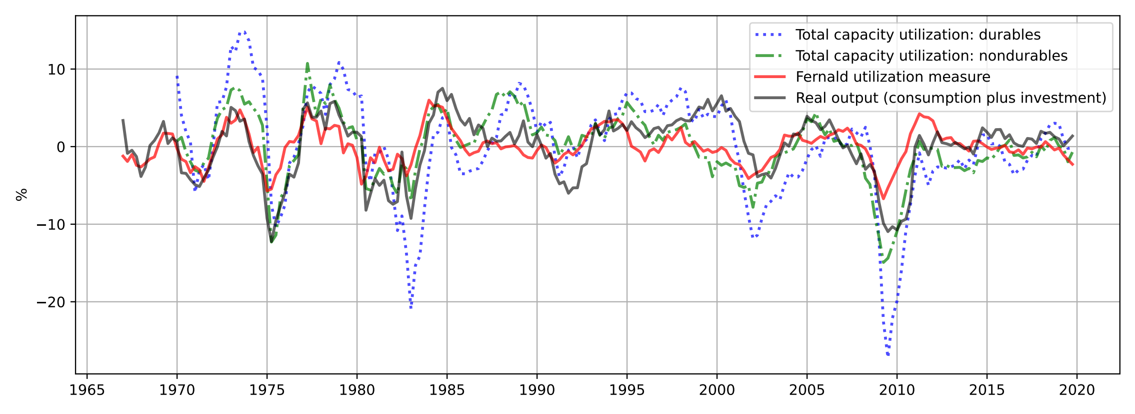

Figure 1: Total capacity utilization in non-durable and durable goods and output, here defined as consumption plus investment. Each underlying series is detrended via the Hamilton regression filter with the four most recent observations 8 quarters in the past ($p = 4, h = 8$).

Related literature

1. Purifying Solow residual:
   Basu, Fernald, and Kimball (2006), Fernald (2014)
2. Goods market frictions and firm productivity
   Moen (1997), Michaillat and Saez (2015), Bai, Rios-Rull, and Storesletten (2024), Huo and Ríos-Rull (2018), Qiu and Ríos-Rull (2022), Petrosky-Nadeau and Wasmer (2015), Bethune, Rocheteau, and Rupert (2015), Borys, Doligalski, and Kopiec (2021), Sun (2024)
3. Sectoral comovement and imperfect intersectoral factor mobility
   Long and Plosser (1983), Christiano and Fitzgerald (1998), Horvath (2000), Katayama and Kim (2018)
4. Total capacity utilization
   Christiano, Eichenbaum, and Trabandt (2016), Qiu and Ríos-Rull (2022)

Production model with shocks and dynamics

Contents

Production technology

- 2 consumption sectors (goods mc and services sc) and an investment sector
- Each uses capital k and labor n to produce output
- Stochastic trend to technology X: growth rate $g_t = X_t/X_{t-1}$ is stationary
- Stationary technology component z_j
- Potential output given capital utilization rate h and fixed cost $\nu_j X$.

$$F_j = z_j f(h_j k_j, n_j) - \nu_j X, \quad j \in \{mc, sc, i\}, \quad (z_{mc} = z_{sc} \equiv z_c)$$

for

$$f(hk, n) = (hk)^{\alpha_k} n^{\alpha_n} X^{1-\alpha_k}, \alpha_k + \alpha_n \leq 1$$

- Set $z_i = z_c z_I$, where z_I is independent of z_c

Matching technology

- **Competitive search**: households shop in markets indexed by price, market tightness, and quantity
- Each market is subject to Cobb-Douglas matching function

$$M_j(D,T) = A_j D^{\phi} T^{1-\phi}$$

where D is aggregate shopping effort and T is the measure of firms (normalize $T = 1$)

- Implied matching rates:

$$\Psi_{jd}(D) = M/D = A_j D^{\phi-1}, \quad \Psi_{jT}(D) = M/T = A_j D^{\phi}$$

so that D describes market tightness

Matching technology

- Once a match is formed, goods are traded at the price $p_j, j \in \{mc, sc, i\}$
- The real quantity of goods purchased given search effort d_j in sector j

$$y_j = \overbrace{d_j}^{\text{search intensity}} \times \underbrace{\Psi_{jd}(D)}_{\text{matching probability}} \times \overbrace{F_j}^{\text{potential output}} \qquad j \in \{mc, sc, i\}$$

Preferences

- Households have GHH preferences over search effort, consumption, and a labor composite

$$u(c, d, n^a, \theta) = \frac{\Gamma^{1-\sigma} - 1}{1 - \sigma}$$

where Γ is a composite parameter with external habit formation:

$$\Gamma = c - haC_{-1} - \theta_d \frac{d^{1+1/\eta}}{1 + 1/\eta} - \theta_n \frac{(n^a)^{1+1/\zeta}}{1 + 1/\zeta}$$

- Aggregate consumption C and total search effort $d = d_{mc} + d_{sc} + \theta_i d_i$
- Preference shifters $\theta = \{\theta_b, \theta_d, \theta_i, \theta_n\}$
- Households shop for investment goods, accumulate capital in each sector, and collect rental income

Consumption aggregator

- Consumption is bundle of goods y_{mc} and services y_{sc}

$$c = [\omega_{mc}^{1-\rho_c} y_{mc}^{\rho_c} + \omega_{sc}^{1-\rho_c} y_{sc}^{\rho_c}]^{1/\rho_c} \tag{1}$$

  such that $\omega_{mc} + \omega_{sc} = 1$
- Price index

$$p_c = \left(\omega_{mc} p_{mc}^{-\rho_c/(1-\rho_c)} + \omega_{sc} p_{sc}^{-\rho_c/(1-\rho_c)}\right)^{-\frac{1-\rho_c}{\rho_c}}$$

- Normalize $p_c = 1$

Imperfect labor mobility across sectors

- Assume imperfect substitutability between labor used in consumption and investment sectors (Horvath (2000) and Katayama and Kim (2018))

$$n^a = \left[\omega^{-\theta} n_c^{1+\theta} + (1-\omega)^{-\theta} n_i^{1+\theta}\right]^{\frac{1}{1+\theta}} \tag{2}$$

- Elasticity of substitution $1/\theta$ measures intersectoral labor mobility
- Induces wage dispersion
- As $\theta \to 0$, $n^a \to n_c + n_i = n$ (perfect mobility benchmark)
- For θ fixed, if $\omega = n_c/n$, then $n^a = n_c + n_i = n$

Differentiated labor and labor unions

- Continuum of monopolistically competitive labor unions in sector j provide services to firms
- Total labor is a CES aggregate of specialized types

$$n_j = \left(\int_0^1 n_j(s)^{1/\mu_j} ds \right)^{\mu_j}$$

- Pay workers W^* per unit and rent to firms at rate $W(s)$
- Rebate earnings to workers

Investment

- Capital law of motion

$$k'_j = (1 - \delta_j(h_j))k_j + [1 - S(i_j/i_{j,-1})]i_j, \quad j \in \{mc, sc, i\}$$

where $i = i_{mc} + i_{sc} + i_i$

- Endogenous capital depreciation (Christiano, Eichenbaum, and Trabandt (2016))

$$\delta^j(h) = \delta^K + \sigma_b(h-1) + \frac{\sigma_{aj}\sigma_b}{2}(h-1)^2, \quad j \in \{mc, sc, i\}, \sigma_{ac} \equiv \sigma_{amc} = \sigma_{asc}$$

$\Rightarrow \sigma_{aj} = \delta''_j(1)/\delta'_j(1)$ is the elasticity of marginal utilization cost wrt h at $h = 1$

- Investment adjustment cost (Christiano, Eichenbaum, and Evans (2005))

$$S(x) = \frac{\Psi_K}{2}(x-1)^2$$

Role of different ingredients

Ingredient	Role	Reference
Capital utilization	Amplification of technology shocks	Christiano, Eichenbaum, and Evans (2005)
Imp. factor mobility	Sectoral comovement	Horvath (2000), Katayama and Kim (2018)
Habit formation	Smooth consumption response	Christiano, Eichenbaum, and Evans (2005)
Inv. Adjustment costs	Hump-shaped investment responses	Christiano, Eichenbaum, and Evans (2005)
Fixed costs	Procyclical productivity	Christiano, Eichenbaum, and Trabandt (2016)
Labor unions	Wage markups	Schmitt-Grohé and Uribe (2012)

Table 2: Compact overview of ingredients and their roles

Timing

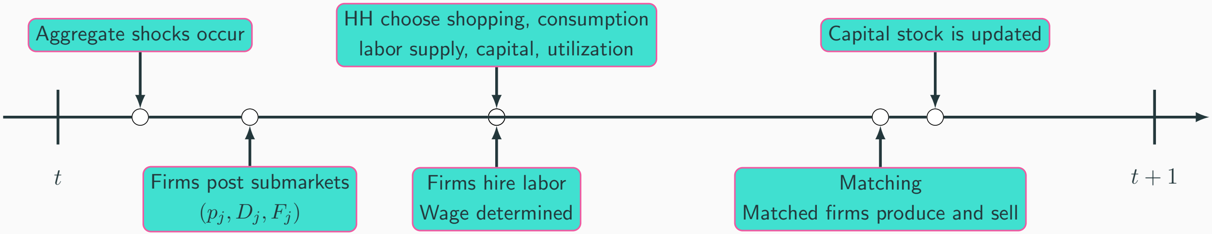

Figure 2: Timing

Households' problem

- Households choose search effort, labor hours, consumption, capital, and utilization rates given markets $(p_j, D_j, F_j), j \in \{mc, sc, i\}$ and the aggregate state of the economy Λ

$$\widehat{V}(\Lambda, \{k_j\}, p, D, F) = \max_{d_j, n_c, n_i, y_j, i_j, k'_j, h'_j} u(y_{mc}, y_{sc}, d, n^a, \theta) + \beta\theta_b \mathbb{E}\{V(\Lambda', \{k'_j\})|\Lambda\}$$

s.t.

$$y_j = d_j \Psi_{jd}(D_j) F_j, \quad j \in \{mc, sc, i\}$$

$$\sum_{j \in \{mc, sc, i\}} y_j p_j = \pi + \sum_{j \in \{mc, sc, i\}} k_j h_j R_j + n_c W_c^* + n_i W_i^*$$

$$k'_j = (1 - \delta_j(h_j))k_j + [1 - S(i_j / i_{j,-1})] i_j, \quad j \in \{mc, sc, i\}$$

subject to endogenous depreciation δ_j, investment adjustment cost S_j, and consumption and labor aggregators (1) and (2)

- The value function is determined by the best market:

$$V(\Lambda, \{k_j\}) = \max_{\{p, D, F\} \in \Phi} \widehat{V}(\Lambda, \{k_j\}, p, D, F)$$

Details of household problem

Optimal shopping effort and demand

- Households equate marginal disutility of shopping effort to marginal utility of consumption in each sector

$$\overbrace{-\frac{u_d}{u_j}}^{\text{MRS}} = \overbrace{\phi\Psi_{jd}(D_j)}^{\Psi'_{jT}(D)} F_j \quad j \in \{mc, sc\} \tag{3}$$

- Two interpretations of (3)
  1. MRS between consumption and shopping effort $(-u_d/u_j)$ equals MRT (increase firm matching probability $\Psi'_{jT}(D)\times$ output sold)
  2. MRS equals HH matching probability multiplied by quantity of output sold and the shopping wedge
- Under GHH preferences, $-u_d/u_j = \theta_d d^{1/\eta}$
- Aggregating,

$$\theta_d D^{1/\eta} = \phi A_j D_j^{\phi-1} F_j, j \in \{mc, sc\}$$

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Optimal shopping effort and demand

- Express value of investment shopping by converting into consumption units using relative price

$$-\frac{u_d}{u_{mc}}\theta_i = \frac{p_i}{p_{mc}}\phi A_i D_i^{\phi-1} F_i$$

- Demand curves for non-durables and services

$$y_j = p_j^{-\xi}\omega_j C \quad j \in \{mc, sc\}$$

given elasticity of substitution $\xi = 1/(1-\rho_c)$

Firms' problem

- A representative firm in sector $j \in \{mc, sc, i\}$ rents capital and hires labor in spot markets
- Firm chooses labor, capital inputs and submarket (p_j, D_j, F_j)
- Submarket must satisfy participation constraint of household

$$\max_{k_j, n_j, p_j, D_j, F_j} p_j A_j D_j^{\phi} F_j - \int_0^1 W_j(s) n_j(s) ds - R_j h_j k_j \quad \text{s.t.}$$

$$z_j f(h_j k_j, n_j) - \nu_j X \geq F_j$$

$$\widehat{V}(\Lambda, \{k_j\}, p_j, D_j, F_j) \geq V(\Lambda, \{k_j\})$$

$$n_j = \left(\int_0^1 n_j(s)^{1/\mu_j} ds \right)^{\mu_j}$$

Details of firm problem

Firm factor demands

$$(1-\phi)\frac{W_j}{p_j} = \alpha_n \frac{A_j D_j^{\phi} z_j f(h_j k_j, n_j)}{n_j} \quad j \in \{mc, sc, i\} \quad W_{mc} = W_{sc}$$

$$(1-\phi)\frac{R_j}{p_j} = \alpha_k \frac{A_j D_j^{\phi} z_j f(h_j k_j, n_j)}{h_j k_j} \quad j \in \{mc, sc, i\}$$

1. Input demand depends positively on shopping effort
2. Matching function elasticity ϕ appears as separate factor
   $\Rightarrow$ Additional output relaxes participation constraint of households and effectively reduces input cost
3. Wage paid by firm is a markup of (variable) wage received by workers

$$W_j = \mu_j W_j^*$$

   with difference $W_j - W_j^*$ rebated to HH as fixed wage

Labor share of income

- Labor share of income is key component to constructing Solow residual
- Define fixed cost share $\nu_j^R = \nu_j X/(z_j f - \nu_j X)$, so that $Y_j = z_j f/(1+\nu_j^R)$
- Write sectoral labor share of income as

$$\frac{W_j n_j}{p_j Y_j} = \frac{\alpha_n (1+\nu_j^R)}{1-\phi}$$

- Provided $\nu_j^R = \nu^R$ for all j, overall labor share of income is

$$\frac{Wn}{Y} = \frac{\alpha_n (1+\nu^R)}{1-\phi}$$

A simple static model

Contents

A simple static model

- Consider simple static model with no investment; homogeneous labor as only input, $F = zn^{\alpha_n}$

Shopping	$\theta_d D^{1/\eta} = \phi C/D$
Consumption	$C = AD^{\phi}F$
Labor demand	$(1-\phi)W = \dfrac{\alpha_n C}{n}$
Labor supply	$\theta_n n^{1/\zeta} = (1-\phi)W$

- Labor share $\tau \equiv Wn/C = \alpha_n/(1-\phi)$ used for computing the Solow residual

$$SR \equiv \frac{C}{n^{\tau}} = AD^{\phi}zn^{\alpha_n - \tau} = AD^{\phi}zn^{-\alpha_n\phi/(1-\phi)}$$

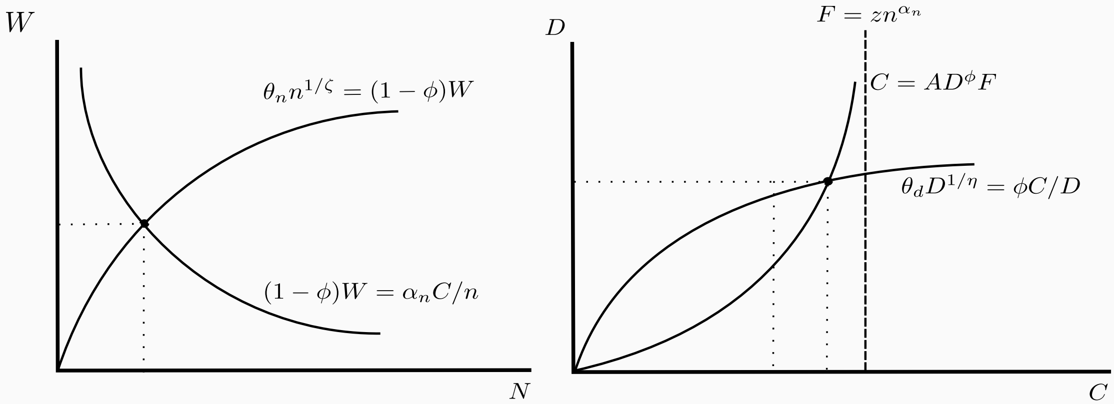

Figure 3: Equilibrium of static model

Demand shock: reduction in θ_d

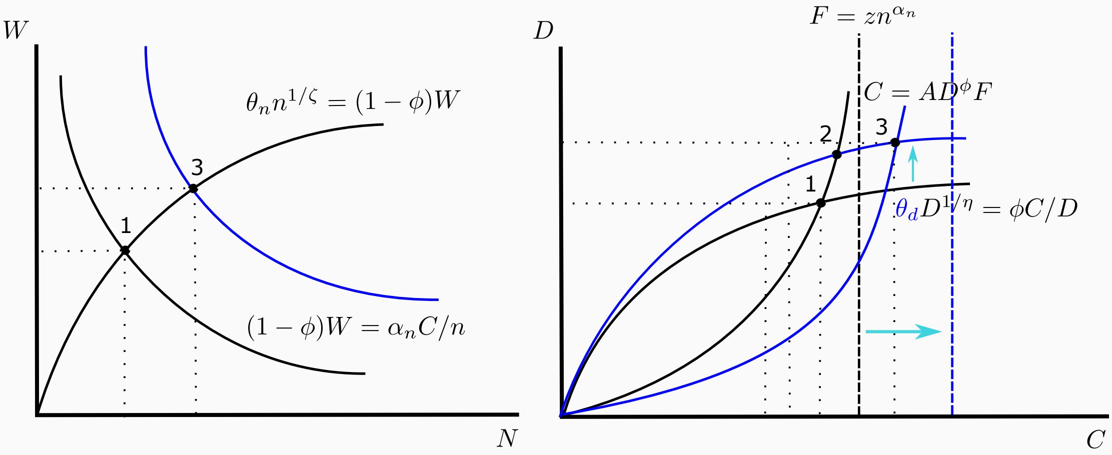

Figure 4: Reduction of shopping disutility in static model

Demand shock: reduction in θ_d

1. Shopping curve shifts upward from lower marginal shopping cost
2. Induces movement along consumption curve from (1) to (2)
3. Firm labor demand shifts rightward, boosting hours and wages
4. More labor hours expand firm potential output, so consumption curve shifts rightward
5. Induces movement along shopping curve from (2) to (3)

Growth rates of Solow residual and capacity utilization

Contents

Sectoral Solow residual: structural decomposition

- Write sectoral Solow residual as

$$SR_{jt} \equiv \frac{Y_{jt}}{k_{jt}^{1-\tau} n_{jt}^{\tau}} = \frac{A_j D_{jt}^{\phi} (z_{jt} h_{jt}^{\alpha_k} X_t^{1-\alpha_k} k_{jt}^{\alpha_k - 1 + \tau} n_{jt}^{\alpha_n - \tau})}{1 + \nu_{jt}^R}$$

given steady-state labor income share τ

- Rewrite using growth rates $dx_t = \Delta \log x_t$

$$dSR_{jt} = \overbrace{\phi dD_{jt}}^{\text{Shopping}} + \underbrace{\alpha_k dh_{jt}}_{\text{Capital utilization}} + \overbrace{dz_{jt} + (1-\alpha_k) dX_t}^{\text{Technology}}$$

$$+ \underbrace{(\alpha_k - 1 + \tau) dk_{jt} + (\alpha_n - \tau) dn_{jt}}_{\text{Input share mismeasurement}} + \overbrace{d(1 + \nu_{jt}^R)}^{\text{Fixed costs}}$$

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Capacity utilization and connection to Solow residual

- Define capacity in sector j following Qiu and Ríos-Rull (2022)

$$cap_j = z_j k_j^{\alpha_k} n_j^{\alpha_n} X^{1-\alpha_k} - \nu_j X$$

- Capacity utilization in sector j is the ratio of output to capacity (stationary measure):

$$util_j \equiv \frac{Y_j}{cap_j} = \frac{A_j D_j^{\phi}(z_j h_j^{\alpha_k} X^{1-\alpha_k} k_j^{\alpha_k} n_j^{\alpha_n} - \nu_j X)}{z_j k_j^{\alpha_k} n_j^{\alpha_n} X^{1-\alpha_k} - \nu_j X}$$

- Capacity utilization in growth rates

$$dutil_{jt} = \phi dD_{jt} + (1+\nu_{ss}^R)\alpha_k dh_{jt}$$

- If $\nu_j = 0$, then Solow residual growth rate simplifies to

$$dSR_{jt}|_{\nu_j=0} = \overbrace{dutil_{jt}}^{\text{Utilization}} + \underbrace{dz_{jt} + (1-\alpha_k)dX_t}_{\text{Technology}} + \overbrace{(\alpha_k - 1 + \tau)dk_{jt} + (\alpha_n - \tau)dn_{jt}}^{\text{Input share mismeasurement}}$$

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Quantitative analysis

Contents

Aggregate measures

- Output

$$Y = C + p_i^{ss} I$$

- Using base-year prices makes results independent of numeraire choice Explanation
- Solow residual and capacity utilization

$$SR = \sum_j \frac{Y_j}{Y} SR_j, \quad util = \sum_j \frac{Y_j}{Y} util_j$$

Equilibrium

BRS as special case

- Model nests Bai, Rios-Rull, and Storesletten (2024) (BRS) by shutting down additional frictions:
  - $ha = 0$
  - $\rho_c = 1$
  - $\nu^R = 0$
  - $\sigma_b \to \infty$
  - $\Psi_K = 0$
  - $\theta = 0$
- Absent fixed costs and variable capital utilization, $util_j = A_j D_j^{\phi}$ and $util = (C/Y)util_c + (I/Y)util_i$

Exercise: role of capacity utilization data in BRS special case

- Fix $\beta = 0.99, \sigma = 2.0$ and Frisch elasticity $\zeta = 0.72$
- Estimate model with same observables as BRS $(Y, I, Y/L, p_i)$ and also with capacity utilization
- Total shock processes $\{\theta_d, \theta_n, g, z, z_I\}$
- In contrast to BRS, estimate ϕ and η, otherwise use same prior distributions

Table 11: Prior distributions

Parameter	Distribution	Mean	Std
ϕ	Beta	0.32	0.20
η	Gamma	0.20	0.15
σ_{e_g}	Inv. Gamma	0.010	0.10
σ_x	Inv. Gamma	0.010	0.10
ρ_g	Beta	0.10	0.050
ρ_x	Beta	0.60	0.20

Table 3: Prior distributions. We use the symbol x as a shorthand for a shock in the set $\{z, z_I, \theta_n, \theta_d\}$.

Role of capacity utilization on parameter estimates

Table 12: Role of capacity utilization on parameter estimates

Parameter	BRS dataset		Add capacity utilization	
	Post. mean	90% HPD interval	Post. mean	90% HPD interval
ϕ	0.0978	[0.0001, 0.205]	0.883	[0.863, 0.906]
η	0.412	[0.282, 0.572]	1.87	[1.86, 1.90]
ρ_d	0.871	[0.775, 0.961]	0.928	[0.914, 0.941]
σ_d	0.0484	[0.0024, 0.0987]	0.0075	[0.0068, 0.0081]

Table 4: Estimation of baseline BRS model with two sets of observable series. The first considers growth rates of output, investment, labor productivity, and the relative price of investment. The second specification also considers total capacity utilization growth.

Goods market frictions and sectoral data

- In this special case, relative shopping effort across sectors equals the relative labor allocation and the relative value of output:

$$\frac{D_c}{D_i} = \frac{n_c}{n_i} = \frac{C}{p_i I}$$

- Stochastic singularity
  - The variables C, I, and p_i are observables in estimation and thus determine n_c/n_i.
  - Trying to use n_c and n_i—or even just their ratio—as observables in estimation would induce stochastic singularity.
- More general model breaks one-for-one link between shopping effort and labor hours using sector-specific wage markup shocks
- Limited factor mobility facilitates sectoral comovement and dampens excessive volatility

Stochastic processes

- The growth rate of the stochastic trend $g_t = X_t/X_{t-1}$ follows an AR(1) process in logs as BRS

$$\log g_t = (1-\rho_g)\log \overline{g} + \rho_g \log g_{t-1} + e_{g,t}$$

  where $e_{g,t} \sim N(0, \sigma_g)$
- Each stationary shock in the set $v = \{\theta_b, \theta_d, \theta_n, \theta_i, z_c, z_I, \mu_c, \mu_i\}$ follows an AR(1) process

$$\log v_t = \rho_v \log v_{t-1} + e_{v,t}$$

  where $e_{v,t} \sim N(0, \sigma_v)$
- Stationarize trending variable by dividing by X_t (X_{t-1} in case of predetermined capital stock K_{jt})

Bayesian estimation: overview

- We use Bayesian estimation to
  1. Quantify parameter uncertainty
  2. Incorporate prior information
  3. Calculate FEVD
  4. Compare role of ingredients in model fit
- Parameter space Θ and data Y
- Sample from posterior distribution combining likelihood and prior

$$P(\Theta|Y) = \frac{L(Y|\Theta)P(\Theta)}{P(Y)}, \quad P(Y) = \int L(Y|\Theta)P(\Theta)d\theta$$

Observable series used for estimation

- Time period: $1964Q1 - 2019Q4$, quarterly frequency
- Use seven observables in growth rates:

$$\mathbf{Y}_t = \begin{bmatrix} dC_t & dI_t & dn_{ct} & dn_i & dutil_{ND,t} & dutil_{D,t} & dp_{it} \end{bmatrix}'$$

- Use sectoral data on output and labor following Katayama and Kim (2018)
- Construct output from sum of private consumption and private investment (as BRS)
- Note that sectoral dataset implicitly targets labor productivity in each sector

Estimation procedure

Calibration

Targets	Value	Parameter	Calibrated value/posterior mean
First group: parameters set exogenously			
Discount factor	0.99	β	0.99
Average growth rate	1.8%	$\overline{g}$	0.45%
Gross wage markup	1.15	μ	1.15
Labor share in consumption	0.8	ω	0.8
Share of services in consumption	0.65	ω_{sc}	0.65
Second group: estimated parameters used for calibration			
Risk aversion	–	σ	1.71
Frisch elasticity	–	ζ	0.93
Elasticity of matching function	–	ϕ	0.75
Elasticity of shopping effort cost	–	η	0.34
Fixed cost share of capacity	–	ν_R	0.24
Habit persistence	–	ha	0.63
Third group: normalizations			
SS output	1	z_{mc}	0.44
Relative price of services	1	z_{sc}	0.63
Relative price of investment	1	z_i	0.37
Fraction time spent working	0.30	θ_n	1.8
Capacity utilization of nondurables	0.81	A_{mc}	2.2
Capacity utilization of services	0.81	A_{sc}	1.4
Capacity utilization of investment sector	0.81	A_i	2.9
Capital utilization rate	1	σ_b	0.033
Fourth group: standard targets			
Investment share of output	0.20	δ	1.4%
Physical capital to output ratio	2.75	α_k	0.28
Labor share of income	0.67	α_n	0.13

Posterior estimates: structural parameters

		Prior			Posterior			
Par	Interpretation	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
σ	Risk aversion	Beta	1.50	0.250	1.71	0.342	1.22	2.23
ha	Habit formation	Beta	0.500	0.200	0.634	0.073	0.510	0.741
ζ	Frisch elast. of labor supply	Gamma	0.720	0.250	0.925	0.162	0.648	1.15
ϕ	Elast. of matching	Beta	0.320	0.200	0.752	0.108	0.579	0.936
η	Elast. shopping disutility	Inv. Gamma	0.200	0.150	0.344	0.110	0.202	0.523
ξ	Cons. elast. of subs.	Inv. Gamma	0.850	0.100	0.865	0.0711	0.760	0.989
ν_R	Fixed cost share	Beta	0.200	0.100	0.244	0.112	0.0913	0.433
σ_{ac}	Depreciation elast:cons	Inv. Gamma	1.000	1.000	1.90	0.365	1.29	2.53
σ_{ai}	Depreciation elast:inv	Inv. Gamma	1.000	1.000	0.444	0.0910	0.298	0.589
Ψ_K	Inv. adj. cost	Gamma	4.000	1.000	8.36	2.14	4.35	11.3
θ	Inv. elast. of labor mobility	Gamma	1.000	0.500	1.06	0.365	0.459	1.67

Posterior and prior density: ϕ and η

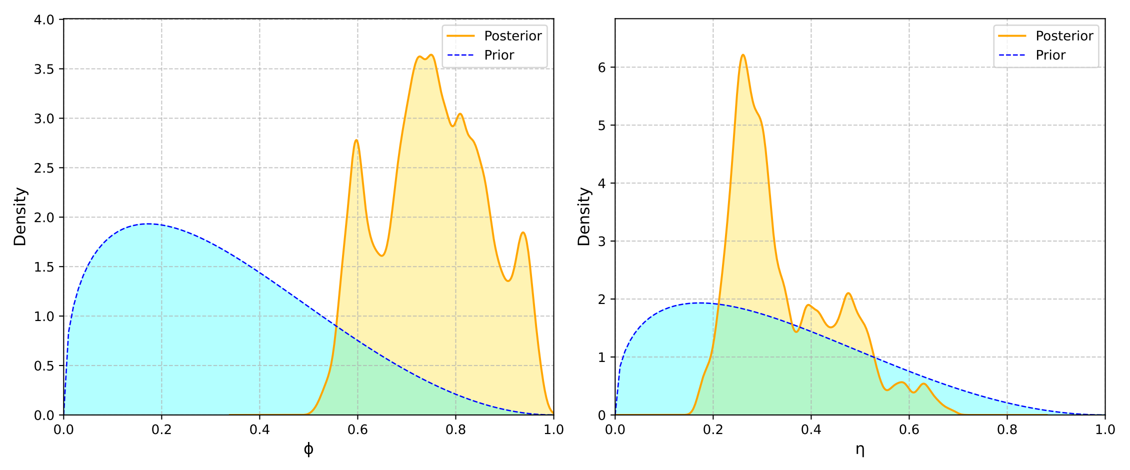

Figure 5: Posterior and prior distributions for matching function elasticity ϕ and shopping disutility parameter η.

Estimation on artificial data (parameter values set to posterior mean)

		Posterior distribution		
Parameter	True value	Median	5%	95%
σ	1.71	1.74	1.52	2.00
ha	0.634	0.651	0.608	0.684
ζ	0.925	1.00	0.866	1.13
ϕ	0.752	0.782	0.738	0.825
η	0.344	0.295	0.220	0.394
ξ	0.866	0.805	0.674	0.934
ν_R	0.244	0.237	0.163	0.312
σ_{ac}	1.90	1.80	1.36	2.26
σ_{ai}	0.444	0.332	0.236	0.440
Ψ_K	8.36	7.09	6.20	9.29
θ	1.06	1.08	0.955	1.22

Table 6: Parameters well identified in exercise using artificial data generated from model evaluated at posterior mean (Schmitt-Grohé and Uribe (2012))

Unconditional forecast error variance decomposition: grouped shocks

Table 7: Unconditional forecast error variance decomposition

	Technology	Labor Supply	Shopping Effort	Discount Factor	Wage Markup
Y	27.78	0.10	71.06	1.01	0.05
SR	42.27	4.29	51.63	0.86	0.95
I	31.31	0.08	63.18	5.41	0.02
p_i	62.39	0.02	36.86	0.30	0.43
n_c	5.71	35.20	53.64	5.12	0.33
n_i	21.10	3.43	55.26	3.44	16.76
$util$	27.15	0.11	71.82	0.90	0.03
D	0.42	0.00	99.56	0.01	0.00
h	22.18	0.08	77.30	0.43	0.01

Table 7: Unconditional forecast error variance decomposition for variables in growth rates. Shocks are grouped in respective categories.

Model comparison

	Data	Baseline	Perfect labor mobility	Common wage markup	Remove: Fixed cost	Remove: VCU	Remove: SDS	Remove: SDS and utilization data
LML	–	4556.7	4529.3	2923.7	4566.8	4473.8	2564.9	–
Δ LML	–	0	-27.4	-1633	10.1	-82.9	-1991.8	–
Posterior mean ϕ	–	0.75	0.39	0.94	0.94	0.36	0.72	0.52
FEVD(Y, SDS)	–	71.06	62.61	5.39	71.16	69.25	–	–
FEVD(SR, SDS)	–	51.63	49.49	4.02	46.76	57.87	–	–
Var($util$)/Var(SR)	–	1.4	0.72	0.32	2.02	0.74	2.21	0.19
std(Y)	0.87	1.6	2.02	7.57	1.38	2.21	207.71	0.64
std($util_{ND}$)	1.26	1.24	1.18	5.08	1.21	1.55	161.65	0.35
std($util_D$)	2.27	3.2	2.69	12.81	3.62	2.43	266.65	1.14
std(n_c)	0.57	0.69	0.67	2.92	0.67	0.89	71.31	0.56
std(n_i)	1.94	2.47	2.77	12.25	2.26	2.01	344.8	1.87
Cor(C, I)	0.54	0.64	0.74	0.09	0.52	0.57	0.999	0.24
Cor($util_{ND}, util_D$)	0.75	0.45	0.76	-0.27	0.29	0.63	0.999	-0.6
Cor(n_c, n_i)	0.87	0.59	0.40	-0.92	0.66	0.27	0.986	0.83
Cor($util_{ND}, util_{ND,-1}$)	0.51	0.31	0.25	0.46	0.23	-0.05	0.999	0.27
Cor($util_D, util_{D,-1}$)	0.55	0.48	0.47	0.37	0.48	0.25	0.999	0.26

Takeaway

1. Baseline model overstates output volatility but otherwise fits sectoral data well
2. Sectoral wage-markup shocks are essential ($\Delta LML = -1633$)
   $\Rightarrow$ shopping-effort ratio is otherwise much more directly tied to labor ratio, and loses flexibility in fitting comovement of utilization
3. Search demand shocks are essential ($\Delta LML = -1992$)
   $\Rightarrow$ Capacity utilization data roughly pins down sectoral shopping efforts–model lacks freedom to jointly match sectoral labor and output and the relative price of investment (close to stochastic singularity)
4. By also removing utilization data, model fits standard sectoral data but misses comovement and volatility of utilization data
5. Imperfect labor mobility significantly helps fit data ($\Delta LML = -27.4$)
6. Removing variable capital utilization is very detrimental ($\Delta LML = -82.9$)
   $\Rightarrow$ Model loses flexibility in explaining utilization and output

Impulse responses under baseline: negative 1 sd shock e_D (shopping disutility shock)

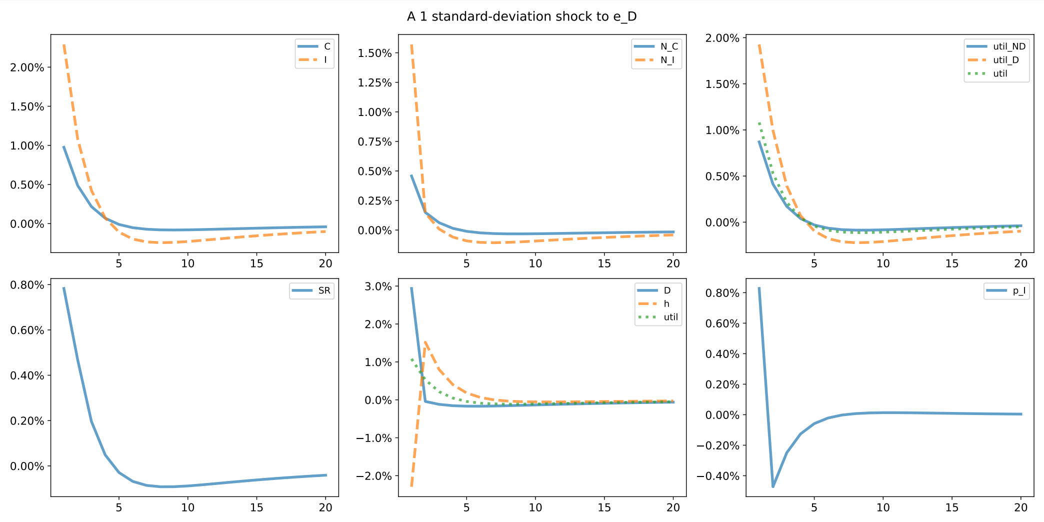

Figure 6: The vertical axis measures response in growth rates.

Impulse responses under baseline: positive 1 sd shock e_z (neutral technology shock)

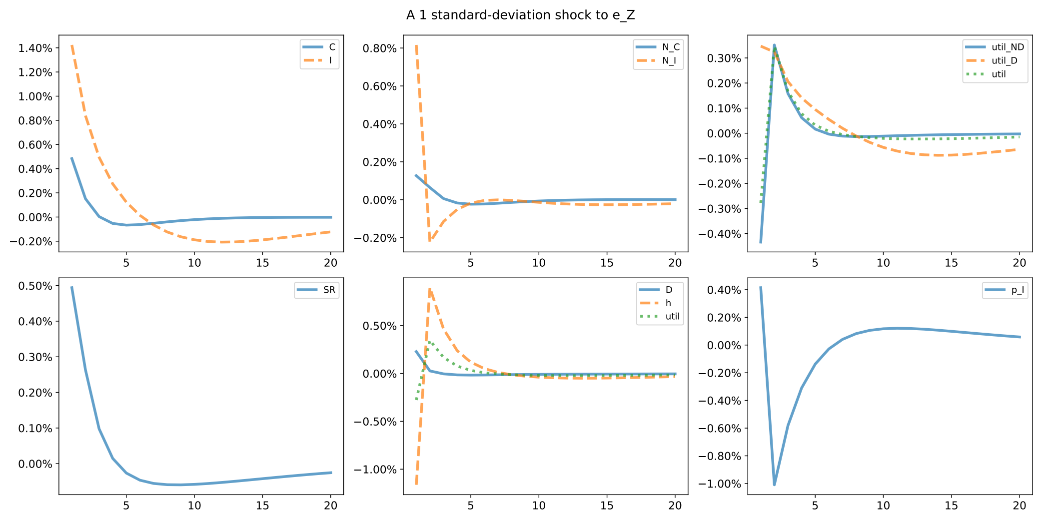

Figure 7: The vertical axis measures response in growth rates.

Impulse responses under baseline: positive 1 sd shock e_b (discount-factor shock)

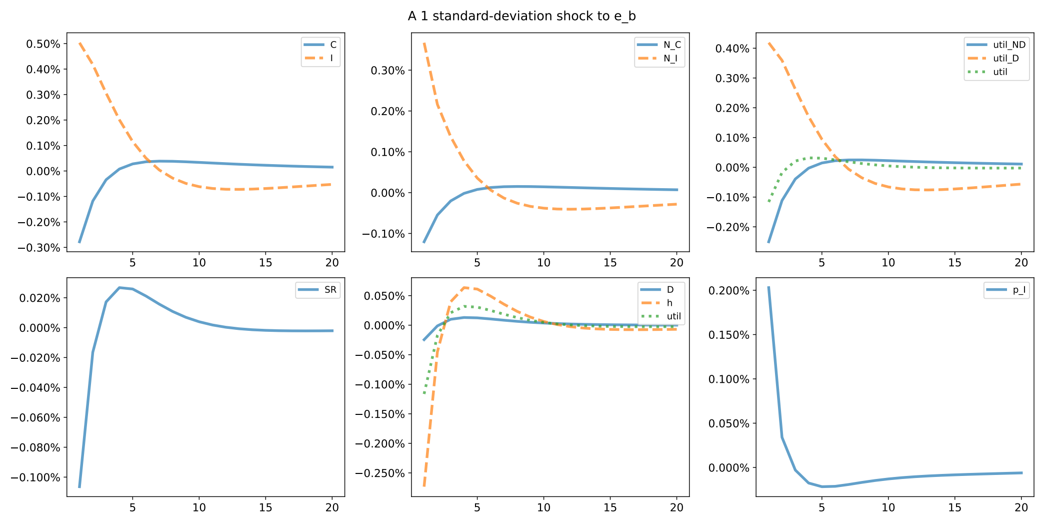

Figure 8: The vertical axis measures response in growth rates.

Conclusion

1. Estimate precise, high value of key parameter ϕ and shopping-effort shocks without relying on shopping time data
2. Search demand shocks explain most of forecast error variance of standard variables and utilization
3. Baseline model fits second moments well other than output
4. Fixed costs are not essential, but removing variable capital utilization prevents fit of utilization autocorrelation
5. Model is incapable of fitting data without search demand shocks or sector-specific wage markups
   5.1 Search effort (e_d) shocks are unique in generating positive comovement between sectoral output, input, and utilization
   5.2 Both technology shocks (e_z and e_g) induce negatively correlated movements in utilization growth$\Rightarrow$ utilization of nondurables falls

Appendix

Contents

Second moments (growth rates)

	SD(x)	STD(x)/STD(Y)	Cor(x, I)	Cor(x, n_I)	Cor(x,x$_{-1}$)
Y	0.87	1.00	0.94	0.70	0.47
C	0.44	0.51	0.54	0.44	0.48
I	2.14	2.46	1.00	0.73	0.41
n_c	0.57	0.66	0.66	0.87	0.67
n_i	1.94	2.23	0.73	1.00	0.64
Y/N	0.64	0.73	0.36	-0.28	0.10
p_i	0.51	0.58	-0.28	-0.22	0.44
$util_d$	2.27	2.61	0.69	0.84	0.55
$util_{nd}$	1.26	1.45	0.61	0.65	0.51

Table 8: Time range: $1964Q1 - 2019Q4$. Each underlying series is expressed in 100 quarterly log deviations. Here output is defined as the sum of consumption and investment.

Back to utilization comovement

Data series

ID	Description	Source
PCND	Personal consumption: non-durable	BEA
PCESV	Personal consumption: services	BEA
HOANBS	Nonfarm business hours worked	BLS
CPIAUCSL	Consumer price index	BLS
GDPC1	Real GDP	BEA
GDPIC1	Real gross private domestic investment	BEA
COMPRNFB	Wages (real compensation per hour)	BLS
CNP160V	Civilian non-institutional population	BLS
GDPDEF	GDP Deflator	BEA
SR	Solow residual	Fernald (2014), FRB of San Francisco
Util	Total capacity utilization	Federal Reserve Board of Governors
SR_{util}	Utilization-adjusted Solow residual	Fernald (2014), FRB of San Francisco

Back to second moments

Construction of variables

Symbol	Description	Construction
C	Nominal consumption	$PCEND + PCESV$
I	Nominal gross private domestic investment	GPDI
Deflator	GDP Deflator	GDPDEF
Pop	Civilian non-institutional population	CNP160V
c	Real per capita consumption	$\frac{C}{Pop * P_c}$
i	Investment	$\frac{I}{Pop * P_i}$
y	Real per capita output	$c + i$
N_c	Labor in consumption sector	Labor in nondurables and services, BLS
N_i	Labor in investment sector	Labor in construction and durables, BLS
N	Aggregate labor	$N_c + N_i$
P_i	Price index: investment goods	$A006RD3Q086SBEA$
P_c	Price index: consumption goods	$DPCERD3Q086SBEA$
p_i	Relative price of investment	P_i/P_c
$util_{ND}$	Total capacity utilization: non-durables	Federal Reserve Board
$util_D$	Total capacity utilization: durables	Federal Reserve Board
SR	Solow residual	Fernald (2014), FRB of San Francisco
SR$_{util}$	Utilization-adjusted Solow residual	Fernald (2014), FRB of San Francisco

More details on construction of sectoral data

- Closely follows Katayama and Kim (2018)
- Construct consumption and investment as follows

$$C_t = \left(\frac{Nondurable(PCND) + Services(PCESV)}{P_c \times CivilianNonstitutionalPopulation(CNP160V)} \right)$$
$$I_t = \left(\frac{Durable(PCDG) + NoresidentialInvestment(PNFI) + ResidentialInvestment(PRFI)}{P_i \times CivilianNoninstitutionalPopulation(CNP160V)} \right)$$

- Use HP-filtered trend for population ($\lambda = 10,000$) to eliminate jumps around census dates
- P_c: combine price indices of nondurable goods (DNDGRG3Q086SBEA) and services (DSERRG3Q086SBEA)
- P_i: use quality-adjusted investment deflator (INVDEV)

More details on construction of sectoral data

- BLS Current Employment Statistics (https://www.bls.gov/ces/data)
- BLS Table B6 contains the number of production and non-supervisory employees by industry
- BLS Table B7 contains average weekly hours of each sector
- We compute total hours for non-durables, services, construction, and durables by multiplying the relevant components of each table
- Construct labor in consumption as sum of non-durables and services
- Construct labor in investment as sum of construction and durables

Households' problem

- Households choose search effort, labor hours, consumption, capital, and utilization rates taking markets $(p_j, D_j, y_j), j \in \{c, i\}$ and the aggregate state of the economy $\Lambda = (\theta, Z, K)$ as given.

$$\widehat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p, D, F) = \max_{d_j, n_c, n_i, y_j, i_j, k'_j, h'_j} u(y_{mc}, y_{sc}, d, n^a, \theta) + \beta\theta_b \mathbb{E}\{V(\Lambda', k'_c, k'_i)|\Lambda\} \quad \text{s.t.}$$

$$y_j = d_j A_j D_j^{\phi-1} F_j, \quad j \in \{mc, sc, i\}$$

$$\sum_j y_j p_j = \pi + \sum_{j \in \{mc, sc, i\}} k_j h_j R_j + n_c W_c + n_i W_i$$

$$k'_j = (1 - \delta_j(h_j))k_j + [1 - S_j(i_j/i_{j,-1})]i_j, \quad j \in \{mc, sc, i\}$$

and the consumption and labor aggregators

- The value function is determined by the best market:

$$V(\Lambda, k_{mc}, k_{sc}, k_i) = \max_{\{p, D, y\} \in \Omega} \widehat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p, D, y)$$

First order conditions

- Let $\gamma_{mc}, \gamma_{sc}, \gamma_i, \lambda, \mu_c, \mu_i$ be the respective Lagrangian multipliers on the constraints
- FOC

$$
\begin{aligned}
&[y_{mc}]: \quad u_{mc} = \gamma_{mc} + \lambda p_{mc} \\
&[y_{sc}]: \quad u_{sc} = \gamma_{sc} + \lambda p_{sc} \\
&[i_c]: \quad -\gamma_i - \lambda p_i + \mu_c \left(1 - S'(x_c)x_c - S(x_c)\right) + \beta\theta_b \mathbb{E}\mu_c' S'(x')(x')^2 = 0 \\
&[i_i]: \quad -\gamma_i - \lambda p_i + \mu_i \left(1 - S'(x_i)x_i - S(x_i)\right) + \beta\theta_b \mathbb{E}\mu' S'(x_i')(x_i')^2 = 0 \\
&[d_j]: \quad u_d = -A_j D_j^{\phi-1} F_j \gamma_j, \quad j \in \{mc, sc\} \\
&[d_i]: \quad u_d \theta_i = -A_i D_i^{\phi-1} F_i \gamma_i \\
&[n_c]: \quad u_n \frac{\partial n^a}{\partial n_c} = -\lambda W_c^* \\
&[n_i]: \quad u_n \frac{\partial n^a}{\partial n_i} = -\lambda W_i^* \\
&[h_j] \quad \delta_h(h_j)\mu_j = \lambda R_j \quad j \in \{mc, sc, i\} \\
&[k_j']: \quad \mu_j = \beta\theta_b \mathbb{E}\left\{\lambda' R_j' h_j' + (1 - \delta_j(h_j'))\mu_j'\right\} \quad j \in \{mc, sc, i\}
\end{aligned}
$$

Envelope conditions

- Consumption

$$\frac{\partial V^j}{\partial p_j} = -\lambda j = -\lambda d_j A_j D_j^{\phi-1} F_j \quad j \in \{mc, sc\} \tag{3}$$

$$\frac{\partial V^j}{\partial D_j} = (\phi - 1) d_j A_j D_j^{\phi-2} F_j (u_j - \lambda p_j) \quad j \in \{mc, sc\} \tag{4}$$

$$\frac{\partial V^j}{\partial F_j} = d_j A_j D_j^{\phi-1} (u_j - \lambda p_j) \quad j \in \{mc, sc\}$$

- Investment

$$\frac{\partial V^i}{\partial p_i} = -\lambda i = -\lambda (d_i A_i D_i^{\phi-1} F_i) \tag{5}$$

$$\frac{\partial V^i}{\partial D_i} = -(\phi - 1) d_i A_i D_i^{\phi-2} F_i \gamma_i \tag{6}$$

$$\frac{\partial V^i}{\partial F_i} = d_i A_i D_i^{\phi-1} \gamma_i$$

Price-tightness tradeoff

- Take ratio of (3) and (4):

$$\frac{\frac{\partial V^j}{\partial p_j}}{\frac{\partial V^j}{\partial D_j}} = -\frac{\lambda D_j}{(\phi - 1)(u_j - \lambda p_j)} \tag{7}$$

- Take ratio of (5) and (6)

$$\frac{\frac{\partial V^i}{\partial p_i}}{\frac{\partial V^i}{\partial D_i}} = -\frac{\lambda D_i}{(\phi - 1)\gamma_i} \tag{8}$$

Back to household problem

Firms' problem

- A representative firm in sector $j \in \{mc, s, i\}$ rents capital and hires labor in spot markets
- Continuum of monopolistically competitive labor unions in sector j sell differentiated services
- Firm chooses inputs and market bundle (p_j, D_j, F_j)
- Submarket must satisfy participation constraint of household

$$\max_{k_j, n_j, p_j, D_j, y_j} p_j A_j D_j^{\phi} F_j - \int_0^1 W_j(s) n_j(s) ds - R_j h_j k_j \quad \text{s.t.}$$

$$\widehat{V}(K, p_j, D_j, F_j) \geq V(K)$$

$$z_j f(h_j k_j, n_j) - \nu_j \geq F_j$$

$$n_j = \left(\int_0^1 n_j(s)^{1/\mu_j} ds \right)^{\mu_j}$$

Conditional labor demand and wage index

- Consider labor cost minimization problem

$$\min_{n_j(s)} \int_0^1 W_j(s) n_j(s) ds \quad \text{s.t.}$$
$$\left(\int_0^1 n_j(s)^{1/\mu_j} dj \right)^{\mu_j} \geq \overline{n}$$

- Take FOC and recognize W_j as Lagrangian multiplier on constraint

$$n_j(s) = \left(\frac{W_j(s)}{W_j} \right)^{-\frac{\mu_j}{\mu_j - 1}} n_j \tag{9}$$

- Wage index for composite labor input in sector j

$$W_j = \left[\int_0^1 W_j(s)^{1/(\mu_j - 1)} ds \right]^{\mu_j - 1}$$

Optimal wage choice of labor union and aggregation

- Problem of labor union

$$\max_{W_j(s)} (W_j(s) - W_j^*) n_j(s) \quad \text{s.t.} \quad (9) \Leftrightarrow$$

$$\max_{W_j(s)} (W_j(s) - W_j^*) \left(\frac{W_j(s)}{W_j} \right)^{-\frac{\mu_j}{\mu_j - 1}} n_j$$

- Labor union in each sector choose

$$W_j(s) = \mu_j W_j^*$$

- Labor unions pay same wage and firms choose identical quantities of labor within j

$$W_j(s) = W_j, n_j(s) = n_j$$

- Labor unions rebate earnings to HH in lump-sum fashion (regard as fixed component to wage)

Firm first order conditions

- Let ι_j and ∇_j be the multipliers on participation constraint and production technology

$$[F_j] \quad \nabla_j = p_j A_j D_j^{\phi} + \iota_j \frac{\partial V^j}{\partial F^j}$$

$$[n_j] \quad W_j = \nabla_j z_j f_n$$

$$[k] \quad h_j R_j = \nabla_j z_j f_k$$

$$[p_j] \quad A_j D_j^{\phi} F_j + \iota_j \frac{\partial V^j}{\partial p_j} = 0 \tag{10}$$

$$[D_j] \quad \phi A_j D_j^{\phi-1} p_j F_j + \iota_j \frac{\partial V^j}{\partial D^j} = 0 \tag{11}$$

Firm problem: finding λ and γ_j

- Take ratio of first order conditions for (10) and (11)

$$\frac{D_j}{\phi p_j} = \frac{\frac{\partial V^j}{\partial p_j}}{\frac{\partial V^j}{\partial D_j}}$$

- Plug in (7)

$$\frac{D_j}{\phi p_j} = -\frac{\lambda D_j}{(\phi - 1)(u_j - \lambda p_j)}$$

- Simplify

$$\lambda \phi p_j = (1 - \phi)(u_j - \lambda p_j) \Rightarrow$$

$$\lambda = u_j(1 - \phi)/p_j$$

so that

$$\gamma_j = \phi u_j$$

Firm problem: finding γ_i

- Take ratio of first order conditions for (10) and (11) for $j = i$:

$$\frac{D_i}{\phi p_i} = \frac{\frac{\partial V^i}{\partial p_i}}{\frac{\partial V^i}{\partial D_i}}$$

- Plug in (8)

$$\frac{D_i}{\phi p_i} = -\frac{\lambda D_i}{(\phi - 1)\gamma_i}$$

- Simplify

$$\begin{aligned}\gamma_i &= \frac{\phi}{1-\phi}\lambda p_i \\ &= \phi \frac{u_j}{p_j} p_i\end{aligned}$$

Simplifying shopping conditions

- Plug in values of γ_j to find

$$-u_d = \phi u_j A_j D_j^{\phi-1}[z_j f(h_j k_j, n_j) - \nu_j] \quad j \in \{m_c, s_c\}$$
$$-u_d \theta_i = \phi \frac{u_{mc} p_i}{p_{mc}} A_i D_i^{\phi-1}[z_i f(h_i k_i, n_i) - \nu_i]$$

- Plug in $\lambda = u_{mc}(1-\phi)/p_{mc}$ to simplify labor-leisure tradeoff

$$u_n \frac{\partial n^a}{\partial n_j} = -\frac{u_{mc}(1-\phi)}{p_{mc}} W_j^* \quad j \in \{c, i\}$$

Demand for non-durables and services

- From the expression for λ we have

$$\frac{u_{mc}}{p_{mc}} = \frac{u_{sc}}{p_{sc}} \Rightarrow \phi = (u_j - \lambda p_j)/u_j$$

- Combine with consumption aggregation and price index to find demand curves

$$Y_j = p_j^{-\xi} \omega_j C \quad j \in \{m_c, s_c\}$$

where $\xi = 1/(1 - \rho_c)$ is the elasticity of substitution.

Tobin's Q

- Solve for value of investment: $j \in \{c, i\}$

$$\lambda p_i + \gamma_i = \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b\mathbb{E}\mu_j'(S'(x_j')(x_j')^2)$$
$$\lambda p_i + \frac{\phi}{1-\phi}\lambda p_i = \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b\mathbb{E}\mu_j'(S'(x_j')(x_j')^2)$$
$$\frac{\lambda p_i}{1-\phi} = \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b\mathbb{E}\mu_j'(S'(x_j')(x_j')^2)$$

- Let $Q_j = \mu_j/\lambda$: relative price of capital in sector j in terms of consumption
- We can rearrange as

$$\frac{p_i}{1-\phi} = Q_j[1 - S'(x_j)x_j - S(x_j)] + \beta\theta_b\mathbb{E}\frac{\lambda'}{\lambda}Q_j'S'(x_j')(x_j')^2$$

Tobin's Q

- Rewrite optimal choice of utilization: $j \in \{mc, sc, i\}$

$$\delta_h(h_j)Q_j = R_j$$

- Euler equation

$$Q_j = \beta\theta_b\mathbb{E}\frac{\lambda'}{\lambda}\left[(1-\delta(h_j'))Q_j' + R_j'h_j'\right] \quad j \in \{mc, sc, i\}$$

Solving for firm multipliers

$$
\begin{aligned}
\iota_j &= \frac{A_j q_j^{\phi} F_j}{\frac{\partial V^j}{\partial p_j}} = \frac{1}{\lambda} \\
\nabla_j &= p_j A_j D_j^{\phi} + \iota_j \frac{\partial V^j}{\partial F^j} \\
&= p_j A_j D_j^{\phi} + \frac{A_j D_j^{\phi} \gamma_j}{\lambda} \\
&= p_j A_j D_j^{\phi} + A_j D_j^{\phi} \frac{\phi}{1-\phi} p_j \\
&= A_j D_j^{\phi} \left(p_j + \frac{\phi}{1-\phi} p_j \right) \\
&= \frac{p_j A_j D_j^{\phi}}{1-\phi}
\end{aligned}
$$

Back to firm problem

Simplified optimality conditions for firm

$$
\begin{aligned}
(1-\phi)\frac{W_c}{p_j} &= A_j(D_j)^{\phi} z_c f_{N_j} \quad j \in \{m_c, s_c\} \\
\frac{W_c}{R_j} &= \frac{f_{N_c}}{f_{K_c}} \\
(1-\phi)\frac{W_i}{p_i} &= A_i(D_i)^{\phi} z_i f_{N_i} \\
\frac{W_i}{R_i} &= \frac{f_{N_i}}{f_{K_i}}
\end{aligned}
$$

Firm factor demands

$$(1-\phi)\frac{W_c}{p_j} = \alpha_n \frac{Y_j + A_j D_j^{\phi} \nu_j}{N_j} \quad j \in \{mc, sc, i\}$$

$$(1-\phi)\frac{R_j}{p_j} = \alpha_k \frac{Y_j + A_j D_j^{\phi} \nu_j}{h_j K_j} \quad j \in \{mc, sc, i\}$$

Summary of equilibrium conditions

$$
\begin{aligned}
\theta_n (n^a)^{1/\nu} \left(\frac{n_c}{n^a}\right)^{\theta} \omega^{-\theta} &= (1-\phi)\frac{W_c}{\mu_c \zeta} \\
\theta_n (n^a)^{1/\nu} \left(\frac{n_i}{n^a}\right)^{\theta} (1-\omega)^{-\theta} &= (1-\phi)\frac{W_i}{\mu_i \zeta} \\
n^a &= \left[\omega^{-\theta} n_c^{1+\theta} + (1-\omega)^{-\theta} n_i^{1+\theta}\right]^{\frac{1}{1+\theta}} \\
\theta_d D^{1/\eta} &= \phi p_j \frac{Y_j}{D_j} \quad j \in \{mc, sc\} \\
\theta_i \theta_d D^{1/\eta} &= \phi p_i \frac{I}{D_i} \\
\frac{p_i}{1-\phi} &= Q_j [1 - S_j'(x_j) x_j - S(x_j)] + \beta \theta_b \mathbb{E} \frac{\lambda'}{\lambda} Q_j' S'(x_j')(x_j')^2 \\
Q_j &= \beta \theta_b \mathbb{E} \frac{\lambda'}{\lambda} \left[(1 - \delta_j(h_j')) Q_j' + R_j' h_j'\right] \quad j \in \{mc, sc, i\}
\end{aligned}
$$

Summary of equilibrium conditions

$$
\begin{aligned}
C &= [\omega_c^{1-\rho_c} Y_{mc}^{\rho_c} + (1-\omega_c)^{1-\rho_c} Y_{sc}^{\rho_c}]^{1/\rho_c} \\
Y_j &= p_j^{-1/(1-\rho_c)} \omega_j C \quad j \in \{mc, sc\} \\
C &= p_{mc} Y_{mc} + p_{sc} Y_{sc} \\
\lambda &= \Gamma^{-\sigma}(1-\phi)
\end{aligned}
$$

Summary of equilibrium conditions

$$
\begin{aligned}
\delta_h(h_j)Q_j &= R_j, \quad j \in \{mc, sc, i\} \\
Y_j &= A_j(D_j)^{\phi}(z_j(h_jK_j)^{\alpha_k}(n_j)^{\alpha_n} - \nu_j) \quad j \in \{mc, sc, i\} \\
I &= I_c + I_i \\
K_j' &= (1-\delta_j(h_j))k_j + [1 - S(x_j)]I_j \quad j \in \{mc, sc, i\} \\
(1-\phi)\frac{W_j}{p_j} &= \alpha_n \frac{Y_j + A_jD_j^{\phi}\nu_j}{N_j} \quad j \in \{mc, sc, i\} \\
\frac{W_j}{R_j} &= \frac{\alpha_n}{\alpha_k}\frac{h_jK_j}{n_j} \quad j \in \{mc, sc, i\}
\end{aligned}
$$

Explanation of numeraire dependence

- Quantity movements may depend on the numeraire in a multisector model
- Consider positive shock to Z^C: relative price of consumption goods falls
- In terms of the investment good, consumption may fall even though actual units purchased rises
- However, if the consumption good were the numeraire, the investment good instead rises in price, so output rises by more
- Reasoning is symmetric with a positive Z^I shock
- Using base-year prices eliminates dependence as by Bai, Rios-Rull, and Storesletten (2024)
- Fisher index also eliminates dependence on base year, but it is equivalent in the case of a first-order approximation.
- See Duernecker, Herrendorf, Valentinyi et al. (2017) for a detailed discussion

Back to mapping

Calibration

Contents

Details: depreciation

- Over sample, the average annual growth rate of output is 1.8%
- Set $\overline{g} = 0.45\%$ (1.8% annual growth)
- Capital accumulation (ignoring adjustment costs)

$$g\widehat{K}' = (1-\delta)\widehat{K} + g\widehat{I}$$

so that in steady state

$$\delta = 1 - \overline{g} + \frac{I}{K}$$

- Let investment share $\kappa = p_i I/Y = 0.2$ and $p_i K/Y = 2.75(4) = 11$
- Hence, $\delta = 0.2/11 - 0.0045 = 1.37\%$

Details: labor share α_n

- Rearrange FOC for labor demand

$$p_j = (1-\phi)\frac{W_j n_j}{\alpha_n A_j (D_j)^\phi F_j}$$

Hence,

$$W_j n_j = \frac{\alpha_n}{1-\phi} p_j Y^j (1+\nu^R)$$

where $\nu^R = \nu_j/(F_j)$ and thus labor share is

$$\frac{\sum W_j n_j}{Y} = \frac{\alpha_n}{1-\phi}\frac{C+p_i I}{Y}(1+\nu^R) = \frac{\alpha_n}{1-\phi}(1+\nu^R)$$

so that $\alpha_n = (1-\phi)$labor share$/(1+\nu^R)$

Details: capital share α_k and deprecation parameter σ_b

- $R_j = R$ in steady state
- Note $\beta(\overline{g})^{-\sigma} = 1/(1+r) \Rightarrow \overline{g} - 1 \approx (r - \rho)/\gamma$
- Implies $\rho \approx r - \gamma\overline{g}$ (so we must have $r \geq \gamma\overline{g}$)
- Steady-state Euler

$$\begin{aligned} Q &= \beta\overline{g}^{-\gamma}[(1-\delta)Q + R] \Rightarrow \\ (1+r)Q &= (1-\delta)Q + R \\ (r+\delta)Q &= R \end{aligned}$$

- Steady-state optimal utilization

$$\sigma_b = \frac{R}{Q} = r + \delta$$

- Combine with steady state Tobin's Q: $p_i/(1-\phi) = Q$ and we find

$$(1-\phi)\frac{R}{p_i} = r + \delta$$

Details: capital share α_k and deprecation parameter σ_b

- Firm optimization yields

$$(1-\phi)\frac{R_j}{p_j} = \alpha_k \frac{Y_j}{K_j}(1+\nu^R)$$

- Note

$$\frac{Y_j}{K_j} = \frac{Y}{K} \quad \forall K$$

  and hence

$$r + \delta = \alpha_k \frac{Y}{K}(1+\nu^R)$$

  so that

$$\alpha_k = \frac{r+\delta}{1+\nu^R}\frac{K}{Y}$$

  Using $r, \delta, K/Y, \nu^R$, we recover $\alpha_k = 0.216$

Details: weight of services ω_{sc}

- We pin down the weight of services ω_{sc} as the empirical measure $S_c = Y_{sc}/C$ and set $S_c = 0.65$.
- The ratio of demand in consumption subsectors implies

$$\frac{Y_{mc}}{Y_{sc}} = \left(\frac{p_{mc}}{p_{sc}}\right)^{-\xi} \frac{\omega_{mc}}{\omega_{sc}}$$

  Multiply each side by p_{mc}/p_{sc}, so that

$$\frac{p_{mc}Y_{mc}}{p_{sc}Y_{sc}} = \left(\frac{p_{mc}}{p_{sc}}\right)^{1-\xi} \frac{\omega_{mc}}{\omega_{sc}}$$

  and plug in S_c, using $\omega_{sc} = S_c$:

$$\left(\frac{1-S_c}{S_c}\right) = \left(\frac{p_{mc}}{p_{sc}}\right)^{1-\xi} \frac{1-S_c}{S_c}$$

  so that $p_{mc} = p_{sc}$

- Given normalization $p_{sc} = 1$, all consumption goods prices equal unity.

Details: matching technology coefficient A_j

- Given $\Psi_j = A_j D_j^{\phi}$, the matching technology coefficient satisfies

$$A_j = \frac{\Psi_j}{D_j^{\phi}}$$

- Need to find D_j for each j

Details: matching technology coefficient A_j

- We first solve for D. Let us sum each side of the shopping optimality condition across sectors:

$$\sum_j D^{1/\eta} D_j = \sum_j \phi p_j Y_j \rightarrow$$
$$D^{\frac{\eta+1}{\eta}} = \phi Y$$

- Given that we choose technology coefficients such that $Y = 1$, we obtain $D = \phi^{\frac{\eta}{\eta+1}}$.

Details: matching technology coefficient A_j

- Consider ratio in shopping optimality conditions between m_c and i:

$$\begin{aligned}\frac{D_{mc}}{D_i} &= \frac{p_{mc}}{p_i}\frac{Y_{mc}}{Y_i} \\ &= (1-\omega_{sc})\frac{1-I/Y}{I/Y}\end{aligned}$$

- Hence,

$$\begin{aligned}D_{mc} &= (1-S_c)(1-I/Y)D \\ D_{sc} &= S_c(1-I/Y)D \\ D_i &= (I/Y)D\end{aligned}$$

Estimation

Contents

Balanced growth and transformation of variables

- Output, consumption, investment, wages, and capital grow at common rate g_t
- Transform each trending variable y_t determined at time t

$$\widehat{y}_t = \frac{y_t}{X_t}$$

  so that $\log \widehat{y}_t$ represents log deviation from stochastic trend

- Capital stock K_t is determined at $t-1$, so we deflate by X_{t-1}

$$\widehat{K}_t = \frac{K_t}{X_{t-1}}$$

- Transform preferences to make shopping stationary

$$\Gamma_t = c_t - haC_{t,-1} - X_t \theta_{dt} \frac{d^{1+1/\eta}}{1+1/\eta} - \theta_{nt} X_t \frac{(n_t^a)^{1+1/\nu}}{1+1/\nu}$$

Equations modified by growth

Observation equations

- Match demeaned growth rates in model to those of data
- Nonstationary series

$$
\begin{aligned}
C_t^{obs} &= \log C_t - \log C_{t-1} + g_t - \overline{g} \\
I_t^{obs} &= \log I_t - \log I_{t-1} + g_t - \overline{g}
\end{aligned}
$$

- Stationary series

$$
\begin{aligned}
N_{jt}^{obs} &= \log N_{jt} - \log N_{j,t-1}, \quad j \in \{c, i\} \\
p_{i,t}^{obs} &= \log p_{i,t} - \log p_{i,t-1} \\
util_{j,t}^{obs} &= \log util_{j,t} - \log util_{j,t-1}
\end{aligned}
$$

Vector of observable variables

Vector of observables

$$= \begin{bmatrix} \Delta \log(C_t) \\ \Delta \log(I_t) \\ \Delta \log(N_{ct}) \\ \Delta \log(N_{it}) \\ \Delta \log(util_{ND,t}) \\ \Delta \log(util_{D,t}) \\ \Delta \log(p_{it}) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Estimation procedure

- Estimate mode of posterior distribution by maximizing log posterior function (combines priors and likelihood)
- Mode is used as initial likelihood
- Use Metropolis-Hastings algorithm to sample posterior distribution and to evaluate marginal likelihood of the model
    - Sample over 1 million draws (discard first 30%)
    - Hessian defines transition probability that generates new proposed draw
- Check convergence and identification (trace plots)

Observation equations

- Match demeaned growth rates in model to those of data
- Nonstationary series

$$C_t^{obs} = \log C_t - \log C_{t-1} + g_t - \overline{g}$$
$$I_t^{obs} = \log I_t - \log I_{t-1} + g_t - \overline{g}$$

- Stationary series

$$N_{jt}^{obs} = \log N_{jt} - \log N_{j,t-1}, \quad j \in \{c, i\}$$
$$p_{i,t}^{obs} = \log p_{i,t} - \log p_{i,t-1}$$
$$util_{j,t}^{obs} = \log util_{j,t} - \log util_{j,t-1}, \quad j \in \{mc, i\}$$

Back to estimation

On the use of growth rates for estimation

- Major macroeconomic series are difference-stationary
- For such data, growth rates preserves all dynamics of a series
- Other filters (such as HP filter/Hamilton filter) extract specific frequencies of time series
- Latter may be reasonable for *description* depending on the notion of business cycle

FEVD: breakdown of search demand shocks

Table 9: Forecast error variance decomposition

	e_d	e_{di}
Y	97.23	2.77
SR	94.26	5.74
I	88.83	11.17
p_i	46.65	53.35
n_c	99.67	0.33
n_i	96.38	3.62
$util$	96.92	3.08
D	99.97	0.03
h	98.77	1.23

Table 9: Contribution of components to forecast error variance decomposition of search shocks.

FEVD: breakdown of technology shocks

Table 10: Forecast error variance decomposition

	e_g	e_Z	e_{zI}
Y	31.68	63.30	5.02
SR	48.24	43.87	7.90
I	3.25	74.14	22.62
p_i	0.14	43.91	55.95
n_c	22.23	75.51	2.26
n_i	6.20	61.70	32.10
$util$	0.64	83.26	16.10
D	10.20	76.28	13.52
h	1.34	89.29	9.36

Table 10: Contribution of components to forecast error variance decomposition of technology shocks.

Back to FEVD

BRS as special case

- Model nests Bai, Rios-Rull, and Storesletten (2024) (BRS) by shutting down additional frictions:
  - $ha = 0$
  - $\rho_c = 1$
  - $\nu^R = 0$
  - $\sigma_b \to \infty$
  - $\Psi_K = 0$
  - $\theta = 0$
- Absent fixed costs and variable capital utilization, $util_j = A_j D_j^{\phi}$ and $util = (C/Y)util_c + (I/Y)util_i$

Exercise: role of capacity utilization data in BRS special case

- Fix $\beta = 0.99, \sigma = 2.0$ and Frisch elasticity $\zeta = 0.72$
- Estimate model with same observables as BRS $(Y, I, Y/L, p_i)$ and also with capacity utilization
- Total shock processes $\{\theta_d, \theta_n, g, z, z_I\}$
- In contrast to BRS, estimate ϕ and η, otherwise use same prior distributions

Table 11: Prior distributions

Parameter	Distribution	Mean	Std
ϕ	Beta	0.32	0.20
η	Gamma	0.20	0.15
σ_{e_g}	Inv. Gamma	0.010	0.10
σ_x	Inv. Gamma	0.010	0.10
ρ_g	Beta	0.10	0.050
ρ_x	Beta	0.60	0.20

Table 11: Prior distributions. We use the symbol x as a shorthand for a shock in the set $\{z, z_I, \theta_n, \theta_d\}$.

Role of capacity utilization on parameter estimates

Table 12: Role of capacity utilization on parameter estimates

Parameter	BRS dataset		Add capacity utilization	
	Post. mean	90% HPD interval	Post. mean	90% HPD interval
ϕ	0.0978	[0.0001, 0.205]	0.883	[0.863, 0.906]
η	0.412	[0.282, 0.572]	1.87	[1.86, 1.90]
ρ_d	0.871	[0.775, 0.961]	0.928	[0.914, 0.941]
σ_d	0.0484	[0.0024, 0.0987]	0.0075	[0.0068, 0.0081]

Table 12: Estimation of baseline BRS model with two sets of observable series. The first considers growth rates of output, investment, labor productivity, and the relative price of investment. The second specification also considers total capacity utilization growth.

Comparison of volatility and variance decomposition

Table 13: Comparison of volatility and variance decomposition

Variable	BRS dataset	Add capacity utilization
Std. dev.		
D	1.54	1.69
$util$	0.15	1.49
FEVD of demand shocks θ_d		
Y	7.73	63.6
Y/N	2.49	27.0
SR	6.14	54.1

Table 13: The first sub-table documents standard deviations of shopping-related variables under two sets of observables. The BRS dataset includes growth rates of output, investment, labor productivity, and the relative price of investment. The second column adds variable total capacity utilization. The second sub-table shows the fraction of the variance decomposition attributable to the demand shock θ_D. See Table 12.

Back to Quantitative analysis

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