Supplemental online appendix

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A. Stochastic singularity in the absence of search demand shocks for a special case of the model

We find numerically in Table 5 that search-based demand shocks are essential to fit the data. Here we show that, in a special case of the model, the absence of search demand shocks actually gives rise to stochastic singularity. That is, an observable series is a deterministic function of other observable series and predetermined variables. Since full-information methods require one to match the entire observed series for some sequence of shocks, this renders estimation impossible under this approach.

Specifically, consider a unitary consumption sector and abstract from fixed costs and investment adjustment costs. Then Equation 19 becomes

$$\frac{p_i}{p_c} = \frac{n_i W_i}{n_j W_j} \frac{C}{I} \tag{1}$$

Absent search demand shocks, Equation 20 simplifies to

$$\frac{D_i}{D_c} = \frac{n_i W_i}{n_c W_c} \tag{2}$$

and hence

$$\frac{D_i}{D_c} = \frac{p_i}{p_c} \frac{I}{C} \tag{3}$$

From (3), the shopping effort ratio is entirely pinned down in terms of observables. Utilization

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in this special case satisfies

$$util_j = A_j D_j^{\phi} h_j^{\alpha_k} \quad j \in \{c, i\} \tag{4}$$

Since, for each j, D_j is pinned down by observables, stochastic singularity arises if h_j is also pinned down by observables.

Recall that optimal utilization has the form $\delta_h^j(h_j)Q_j = R_j$ for $Q_j = p_j/(1-\phi)$. Moreover, we can express the rental of capital as $R_j = \alpha_k Y_j/(h_j K_j)$ and hence

$$\delta_h^j(h_j)\frac{p_j}{1-\phi} = \alpha_k \frac{Y_j}{h_j k_j} \tag{5}$$

so that h_j is a function of observables and predetermined capital. Consequently, using (4), utilization in each sector is a function of other observables, and there is stochastic singularity.

B. Equilibrium of simplified model

Given initial states $\{k_{c0}, k_{i0}\}$ and $\{g_0, \theta_{d0}, \theta_{n0}, z_{c0}, z_{i0}\}$, an equilibrium is a sequence of prices $\{p_{it}, R_{ct}, R_{it}, W_t\}_{t=0}^{\infty}$ and quantities $\{k_{ct}, k_{it}, k_t, C_t, I_t, D_{ct}, D_{it}, D_t, n_{ct}, n_{it}, n_t, g_t, \theta_{dt}, \theta_{nt}, z_{ct}, z_{It}\}_{t=0}^{\infty}$ which solve the following system given the realization of shocks $\{e_{gt}, e_{vt}\}_{t=0}^{\infty}$:

$$\theta_{nt} n_t^{1/\nu} = (1-\phi) W_t \tag{6}$$

$$\theta_{dt} D_t^{1/\eta} = \phi \frac{C_t}{D_{ct}} \tag{7}$$

$$\theta_{dt} D_t^{1/\eta} = \phi p_{it} \frac{I_t}{D_{it}} \tag{8}$$

$$\Gamma_t = C_t - \theta_{dt} \frac{D_t^{1+1/\eta}}{1+1/\eta} - \theta_{nt} \frac{(n_t)^{1+1/\zeta}}{1+1/\zeta} \tag{9}$$

$$\Gamma_t^{-\sigma} p_{it} = \beta \mathbb{E}\left\{[(1-\phi) R_{c,t+1} + p_{i,t+1}(1-\delta)](\Gamma_{t+1} g_{t+1})^{-\sigma}\right\} \tag{10}$$

$$\mathbb{E}(R_{c,t+1} - R_{i,t+1}) = 0 \tag{11}$$

$$C_t = A_c (D_{ct})^{\phi} z_{ct} g_t^{-\alpha_k} k_{ct}^{\alpha_k} n_{ct}^{\alpha_n} \tag{12}$$

$$I_t = A_i (D_{it})^{\phi} z_{it} g_t^{-\alpha_k} k_{it}^{\alpha_k} n_{it}^{\alpha_n} \tag{13}$$

$$I_t g_t = (k_{c,t+1} + k_{i,t+1}) g_t - (1-\delta)(k_{ct} + k_{it}) \tag{14}$$

$$(1-\phi) \frac{W_t}{p_t} = \alpha_n \frac{C_t}{n_{ct}} \quad j \in \{c, i\}, \quad \text{with} \quad p_{ct} = 1 \tag{15}$$

$$\frac{W_t}{R_{jt}} = \frac{\alpha_n}{\alpha_k} \frac{k_{jt}}{n_{jt}} \quad j \in \{c, i\} \tag{16}$$

$$n_t = n_{ct} + n_{it}, k_t = k_{ct} + k_{it}, D_t = D_{ct} + D_{it} \tag{17}$$

$$\log g_t = (1-\rho_g)\overline{g} + \rho_g \log g_{t-1} + e_{gt} \tag{18}$$

$$\log v_t = \rho_v \log v_{t-1} + e_{v,t}, v \in \{\theta_d, \theta_n, z_c, z_I\} \tag{19}$$

$$\log z_{it} = \log z_{ct} + \log z_{It} \tag{20}$$

C. Equilibrium of baseline model

Given initial states $\{k_{mc0}, k_{sc0}, k_0\}$ and $\{g_0, \theta_{b0}, \theta_{d0}, \theta_{i0}, \theta_{n0}, z_{c0}, z_{I0}, \mu_{c0}, \mu_{i0}\}$, an equilibrium is a sequence of prices $\{p_{it}, R_{jt}, Q_{jt}, W_{ct}, W_{it}\}_{t=0}^{\infty}$ and quantities $\{k_{jt}, i_{jt}, Y_{jt}, C_t, D_{jt}, n_t^a, n_{jt}, n_{ct}, n_t, g_t, \theta_{bt}, \theta_{dt}, \theta_{it}, \theta_{nt}, z_{ct}, z_{It}, \mu_{ct}, \mu_{it}\}_{t=0}^{\infty}$ for $j \in \{mc, sc, i\}$ that solves the following system given the realization of shocks $\{e_{gt}, e_{vt}\}_{t=0}^{\infty}$:

$$\theta_n (n_t^a)^{1/\nu} \left(\frac{n_{ct}}{n_t^a}\right)^{\varepsilon} \omega^{-\varepsilon} = (1-\phi)\frac{W_{ct}}{\mu_{ct}} \tag{21}$$

$$\theta_n (n_t^a)^{1/\nu} \left(\frac{n_{it}}{n_t^a}\right)^{\varepsilon} (1-\omega)^{-\varepsilon} = (1-\phi)\frac{W_{it}}{\mu_{it}} \tag{22}$$

$$n_t^a = \left[\omega^{-\varepsilon} n_{ct}^{1+\varepsilon} + (1-\omega)^{-\varepsilon} n_{it}^{1+\varepsilon}\right]^{\frac{1}{1+\varepsilon}} \tag{23}$$

$$\Gamma_t = C_t - \theta_{dt}\frac{D_t^{1+1/\eta}}{1+1/\eta} - \theta_{nt}\frac{(n_t)^{1+1/\zeta}}{1+1/\zeta} \tag{24}$$

$$\theta_{dt} D_t^{1/\eta} = \phi p_{jt}\frac{Y_{jt}}{D_{jt}} \quad j \in \{mc, sc\} \tag{25}$$

$$\theta_{it}\theta_{dt} D_t^{1/\eta} = \phi p_{it}\frac{I_t}{D_{it}} \tag{26}$$

$$\frac{p_{it}}{1-\phi} = Q_{jt}[1 - S'(x_{jt})x_{jt} - S(x_{jt})] + \beta\theta_b \mathbb{E}_t \left(\frac{\Gamma_{t+1}}{\Gamma_t}\right)^{-\sigma} g_{t+1}^{-\sigma} Q_{j,t+1} S'(x_{j,t+1})(x_{j,t+1})^2 \quad j \in \{mc, sc, i\} \tag{27}$$

$$Q_{jt} = \beta\theta_{bt}\mathbb{E}t \left(\frac{\Gamma_{t+1}}{\Gamma_t}\right)^{-\sigma} g_{t+1}^{-\sigma}\left[(1-\delta_j(h_{j,t+1}))Q_{j,t+1} + R_{j,t+1}h_{j,t+1}\right] \qquad j \in \{mc, sc, i\} \tag{28}$$

$$C_t = [\omega_c^{1-\rho_c} Y_{mc,t}^{\rho_c} + (1-\omega_c)^{1-\rho_c} Y_{sc,t}^{\rho_c}]^{1/\rho_c} \tag{29}$$

$$Y_{jt} = p_{jt}^{-1/(1-\rho_c)} \omega_j C_t \quad j \in \{mc, sc, i\} \tag{30}$$

$$C_t = p_{mc,t} Y_{mc,t} + p_{sc,t} Y_{sc,t} \tag{31}$$

$$\delta_h(h_{jt}) Q_{jt} = R_{jt}, \quad j \in mc, sc, i \tag{32}$$

$$Y_{jt} = A_j (D_{jt})^{\phi} (z_{jt} g_t^{-\alpha_k} (h_{jt} k_{jt})^{\alpha_k} (N_{jt})^{\alpha_n} - \nu_j) \quad j \in \{mc, sc, i\} \tag{33}$$

$$k_{j,t+1} g_t = (1-\delta_j(h_{jt}))k_{jt} + [1 - S(x_{jt})] I_{jt} g_t \quad j \in \{mc, sc, i\} \tag{34}$$

$$(1-\phi)\frac{W_{jt}}{p_{jt}} = \alpha_n \frac{Y_{jt} + A_j D_{jt}^{\phi}\nu_j}{n_{jt}} \quad j \in \{mc, sc, i\} \tag{35}$$

$$\frac{W_{jt}}{R_{jt}} = \frac{\alpha_n}{\alpha_k}\frac{h_{jt}k_{jt}}{n_{jt}} \quad j \in \{mc, sc, i\} \tag{36}$$

$$n_{ct} = n_{mct} + n_{sct}, n_t = n_{ct} + n_{it}, D_t = D_{mct} + D_{sct} + D_{it} \tag{37}$$

$$k_t = k_{mct} + k_{sct} + k_{it}, I_t = I_{mct} + I_{sct} + I_{it} \tag{38}$$

$$\log g_t = (1-\rho_g)\log \overline{g} + \rho_g \log g_{t-1} + e_{g,t} \tag{39}$$

$$\log v_t = \rho_v \log v_{t-1} + e_{v,t}, \quad v \in \{\theta_b, \theta_d, \theta_n, \theta_i, z_c, z_I, \mu_c, \mu_i\} \tag{40}$$

D. Convergence diagnostics

Figure 1 presents the multivariate convergence diagnostics from the Metropolis Hastings. The top subplot (Interval) shows the Brooks and Gelman (1998) convergence diagnostics for the 80% interval. The blue line shows the 80% interval based on pooled draws from all sequences, while the red line shows the mean interval range based on draws of the individual sequences. The second and third subplots (m2 and m3, respectively) show an estimate of the same statistics for the squared and cubed absolute deviations from the pooled and within-sample mean, respectively. We can visually assess convergence in terms of the blue and red lines stabilizing horizontally and being close to each other.

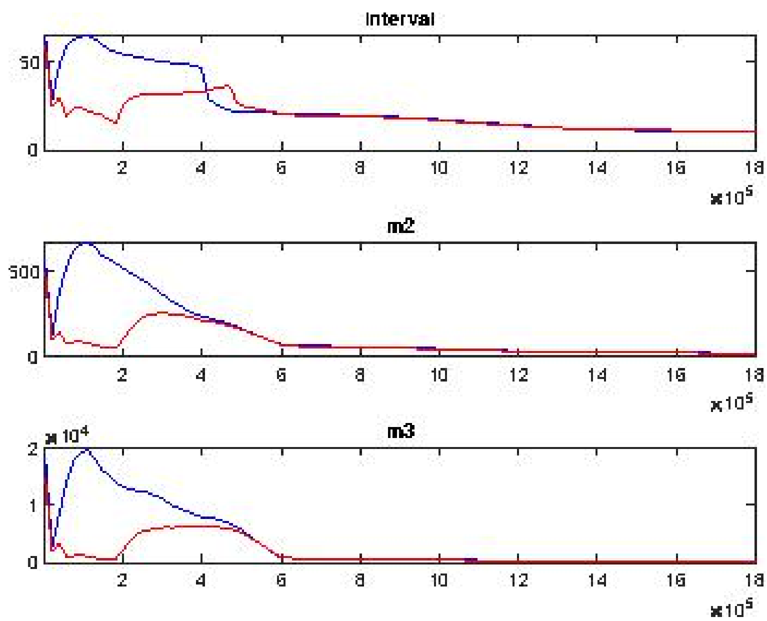

Figure 1: Multivariate convergence diagnostics for the Metropolis-Hastings. The first, second and third rows are respectively the criteria based on the eighty percent interval, the second and third moments. The different parameters are aggregated using the posterior kernel.

E. The forecast error variance decomposition for specific demand and technology shocks

Here we decompose the variance decomposition of demand and technology shocks. The main takeaway from Table 1 is that neutral search demand shocks dominate the forecast error variance of all variables except for the relative price of investment. In particular, it accounts for over 96% of the demand component of utilization.

Table 1: Forecast error variance decomposition

	e_d	e_{di}
Y	97.23	2.77
SR	94.26	5.74
I	88.83	11.17
p_i	46.65	53.35
n_c	99.67	0.33
n_i	96.38	3.62
$util$	96.92	3.08
D	99.97	0.03
h	98.77	1.23

Table 1: Contribution of components to forecast error variance decomposition of search shocks.

In a similar vein, Table 2, dissects the various constituent elements of technology shocks. Stationary neutral technology shocks e_z are by far the most important overall. However, permanent technology shocks are relatively important for output and especially the Solow residual. Investment-specific technology shocks are, unsurprisingly, important for investment, its relative price, and labor in the investment sector. From both tables it is clear that each is important at explaining at least some aspect of business cycle fluctuations.

Table 2: Forecast error variance decomposition

	e_g	e_z	e_{zI}
Y	31.68	63.30	5.02
SR	48.24	43.87	7.90
I	3.25	74.14	22.62
p_i	0.14	43.91	55.95
n_c	22.23	75.51	2.26
n_i	6.20	61.70	32.10
$util$	0.64	83.26	16.10
D	10.20	76.28	13.52
h	1.34	89.29	9.36

Table 2: Contribution of components to forecast error variance decomposition of technology shocks.

F. Estimation on artificial data and identification of parameters

To assess the identifiability of key parameters, we conduct an analysis employing artificial data inspired by Schmitt-Grohé and Uribe (2012). This involves setting the parameters at their mean values and following the calibration strategy outlined in Section 6 and Appendix D. We generate an artificial dataset comprising 223 observations for each observable variable. Subsequently, we estimate the model using this artificial data, employing the same estimation techniques and prior distributions as in the baseline model. Compared to the posterior-prior informativeness criterion considered in the main text, this simulation-based parameter recovery method focuses on identification properties of the model itself rather than the actual data.

Table 3 plots the true value used in generating the artificial data alongside the 5th, 50th, and 95th percentiles of the posterior distribution for each parameter value. We find that the highest posterior density intervals typically contain the true parameter value, often toward the center. In particular, the posterior median for ϕ, 0.911, is very close to 0.913. The persistence parameters of demand shocks ρ_d and ρ_{di} are well-identified, though the posterior probability band for e_d does not capture the true value. There is also excellent identification of $ha, \sigma, \varepsilon, \xi$, and ν^R. The posterior probability bands also contain the true values of the persistence and conditional standard deviation of technology shocks. Estimates are highly diffuse, however, for the conditional standard deviation of wage markup shocks $e_{\mu C}$.

Table 3: Estimation on artificial data

		Posterior distribution		
Parameter	True value	Median	5%	95%
σ	1.58	1.70	1.24	1.85
ha	0.736	0.714	0.689	0.746
ν	1.24	1.29	1.09	1.50
ϕ	0.913	0.911	0.872	0.955
ζ	0.224	0.241	0.203	0.276
ξ	0.882	0.809	0.717	0.892
ν^R	0.0943	0.118	0.0694	0.21
σ_{ac}	1.76	2.04	1.69	2.47
σ_{ai}	0.441	0.289	0.216	0.374
Ψ_K	12.5	7.78	7.26	8.30
ε	1.46	1.53	1.38	1.69
ρ_g	0.516	0.333	0.221	0.387
ρ_z	0.793	0.748	0.648	0.850
ρ_{zi}	0.848	0.820	0.769	0.870
ρ_n	0.989	0.911	0.851	0.969
ρ_d	0.906	0.885	0.844	0.927
ρ_{di}	0.982	0.933	0.883	0.983
ρ_b	0.911	0.921	0.878	0.958
$\rho_{\mu c}$	0.759	0.931	0.835	0.979
$\rho_{\mu i}$	0.977	0.959	0.935	0.986
e_g	0.00437	0.00407	0.00362	0.00455
e_Z	0.00941	0.00828	0.00738	0.00915

e_{zi}	0.0194	0.0185	0.0165	0.0203
e_n	0.00613	0.00597	0.00456	0.00727
e_d	0.136	0.102	0.0895	0.112
e_{di}	0.0151	0.0145	0.0132	0.0158
e_b	0.0154	0.0100	0.00527	0.0156
$e_{\mu c}$	0.000747	0.00170	0.0001	0.00376
$e_{\mu i}$	0.0273	0.0267	0.0242	0.0292

Table 3: We generate artificial data from the model with parameter values equal to the posterior mean of the Bayesian estimation on the actual data, in tandem with the calibration strategy. We then use this artificial data as observables in estimation. The posterior median, 5th percentile, and 95th percentile from the posterior distribution are compared alongside the true values.

G. Shopping costs in the form of expenditure

Michaillat and Saez (2015) also use matching frictions in the goods market and emphasizes the impact of aggregate-demand shocks on output and employment. At first glance, it is difficult to compare the two settings because Michaillat and Saez (2015) specify the matching frictions differently, formalize matching costs in terms of expenditure rather than disutility, and also incorporate money demand via money in the utility. Accordingly, we represent matching costs in terms of expenditures in a static form of BRS and show that the same key logic holds. However, the labor share of income turns out to be different since expenditure shows up in the national income accounts, but effort does not.

As in the static model in the main text, each firm has a location production function $F = zn^{\alpha_n}$ using just labor. Each unit of search requires an expenditure ρ. In terms of national income accounting, these expenditures are part of consumption, but they yield no utility to households. The remaining part of consumption, c^e, does directly yield utility.

Household preferences take the form $u(c^e, n) = U(\Gamma)$ where U is increasing, strictly concave, and differentiable

$$\Gamma = c^e - \theta_n \frac{n^{1+1/\zeta}}{1+1/\zeta} \tag{41}$$

Thus, there are zero wealth effects on labor supply (GHH).

The link between effective consumption and overall consumption satisfies

$$c^e = C - d\rho \tag{42}$$

$$= d(\Psi_d F - \rho) \tag{43}$$

The necessary units of shopping to consume one service are $1/(\Psi_d F - \rho)$. The associated expenditures are thus

$$T(D) = \frac{\rho}{\Psi_d F - \rho} \tag{44}$$

The expression for T in (44) differs from the analogue in Michaillat and Saez (2015) only by the fact that the Ψ_d is multiplied by capacity F, which is a consequence of one unit traded per match in their setup.

A household who chooses a particular submarket (p, D) has expenditure $pc^e(1+T(D)) = pC$ and associated income $\pi + nW$, where π denotes firms' profits.

The problem of the household in submarket (p, D) is

$$\max u(c^e, n) \quad s.t. \tag{45}$$

$$pc^e(1 + T(D)) = \pi + nW \tag{46}$$

The first order conditions with respect to c and n yield the following labor-leisure or labor supply condition:

$$\theta_n n^{1/\zeta} = \frac{W/p}{1 + T(D)} \tag{47}$$

The search wedge $1/(1 + T(D))$ reduces the return to working, analogous to a consumption tax or labor income tax.

We next solve the problem of the firm. To keep customers from deviating to another submarket, it must post a combination of price and tightness (p, D) such that $p(1+T(D)) \leq H$ for some H. The problem is

$$\max_{n,p,D} p\Psi_T(D)zn^{\alpha_n} - nW \quad s.t. \tag{48}$$

$$p(1+T(D)) \leq H \tag{49}$$

The first order condition for n is

$$\alpha_n \frac{\Psi_T F}{n} = W$$

Aggregate consumption satisfies $C = \Psi_T F$, so that $nW/C = \alpha_n$. Hence, the labor share of income is α_n. By contrast, if the matching costs were in terms of disutility, then the corresponding labor share of income would be $\alpha_n/(1-\phi)$.

The problem over the price-tightness pair (p, D) can be simplified by substituting for the constraint in the objective as

$$\frac{\Psi_T(D)}{1+T(D)} = \frac{\Psi_T}{\Psi_D}(\Psi_d F - \rho) = \frac{D}{F}(AD^{\phi-1}F - \rho)$$

Differentiating with respect to D yields

$$\rho = \phi\Psi_D F \tag{50}$$

or, in closed form,

$$D = \left(\frac{\phi Azn^{\alpha_n}}{\rho}\right)^{1/(1-\phi)} \tag{51}$$

Notice that (51) depends not only on both the parameters of matching technology ϕ, A and cost ρ but also on z and n.

Thus, we normalize $p = 1$ and define equilibrium as a tuple (D, C, c^e, n, W) satisfying

$$\rho = \phi \Psi_D \tag{52}$$

$$C = AD^{\phi} z n^{\alpha_n} \tag{53}$$

$$c^e = \frac{C}{1 + T(D)} \tag{54}$$

$$W = \frac{\alpha_n C}{n} \tag{55}$$

$$\theta_n n^{1/\zeta} = \frac{W}{1 + T(D)} \tag{56}$$

Compared to the baseline setup, the wedge on labor supply is now $1/(1 + T(D))$ instead of $1 - \phi$ and the labor share of income is α_n. Moreover, the cost of shopping is linear, which is analogous to letting $\eta \to \infty$ in the BRS specification.

A key difference in the labor share of income is that purchased shopping services are still part of GDP. Thus, the Solow residual is $SR = C/n^{\alpha_n} = AD^{\phi} z$. Both matching frictions and technology enter into GDP, but, unlike BRS, there is no input share mismeasurement.

Michaillat and Saez (2015) argue that the effect of aggregate demand shocks on output and employment depends on sticky prices. The reason is that the demand shocks they consider–consumption preference or money supply–do not affect *efficient* level of market tightness. Under competitive search, tightness is necessarily at the efficient level, so some deviation would thus be necessary for such demand shocks to matter.

However, under the matching setup considered here, the efficient level of market tightness also depends on labor hours and technology. It follows that any demand shock that affects labor demand also raises D and the Solow residual. In the current bare-bones setup, a reduction in θ_n stimulates labor demand, which raises shopping and tightness. Additionally, we included money as Michaillat and Saez (2015), then a consumption preference shock or shock to the level of money supply would also affect labor and hence tightness.

In general, the influence of labor hours on the efficient level of tightness holds provided that the expenditure ρ does not scale one-for-one with capacity. If the cost of a shopping were ρF instead of ρ, then we would instead have $T = \rho/(\Psi_d - \rho)$ and D would be determined by $\rho = \phi \Psi_d$. The efficient level of tightness would just depend on ϕ, A, and ρ. We believe it plausible a priori that shopping expenditure costs scale less than one-for-one with firm capacity, though of course parsing these micro-level features require more granular data and research.

H. Related measures of capacity utilization for other countries

Country	Sectors Covered	Survey/ Calc.?	How Question is Framed / Method	Source
Canada	Manufacturing, mining, utilities, construction	Survey	"At what percentage of your production capacity are you currently operating?"	Statistics Canada
Euro Area	Manufacturing	Survey	"At what capacity is your company currently operating (as a percentage of full capacity)?"	European Commission
UK	Manufacturing, services	Survey	"What is the current rate of capacity utilization in your business?" (firms give a percentage)	Office for National Statistics / CBI
Japan	Manufacturing, mining	Calculated	Based on indices of industrial production and capacity, using statistical/engineering estimates	Ministry of Economy, Trade and Industry (METI)
South Korea	Manufacturing	Survey	Firms are surveyed: "At what % of capacity are you currently operating?"	Statistics Korea (KOSTAT)
Russia	Manufacturing	Survey	Firms report their current use of production capacity as % of "normal/full" capacity	Rosstat
China	Manufacturing	Survey	Firms asked: "What is the current utilization rate of your production capacity?"	National Bureau of Statistics of China (NBSC)

Table 4: International Capacity Utilization Measures

References

Brooks, S. P., and A. Gelman (1998): "General methods for monitoring convergence of iterative simulations," *Journal of computational and graphical statistics*, 7(4), 434–455.

Michaillat, P., and E. Saez (2015): "Aggregate demand, idle time, and unemployment," *The Quarterly Journal of Economics*, 130(2), 507–569.

Schmitt-Grohé, S., and M. Uribe (2012): "What's news in business cycles," *Econometrica*, 80(6), 2733–2764.