Productive demand, sectoral comovement, and total capacity utilization

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Abstract

We investigate business cycle fluctuations in a three-sector model where demand shocks influence the Solow residual and estimate it using Bayesian techniques. Our novel identification strategy uses capacity utilization data from both nondurable and durable goods sectors to identify key parameters of goods market frictions. In our simplified setting, incorporating capacity utilization data reveals that goods market frictions and demand shocks play a more significant role than indicated by an estimation which only uses conventional macroeconomic variables. In our general setting, shocks to shopping effort account for the majority of the forecast error variance in output, the Solow residual, and utilization. Furthermore, search demand shocks and sector-specific wage markup shocks prove essential for inducing positive comovement of utilization data and fitting sectoral data overall.

Keywords: goods market frictions, capacity utilization, sectoral comovement, endogeneity of Solow residual, Bayesian estimation

JEL Classification: D10; E21; E22; E32; E37

1. Introduction

Macroeconometric work has established that the Solow residual is not a pure measure of technology. Evans (1992) shows that money, interest rates, and government expenditure

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Granger causes the Solow residual. Basu, Fernald, and Kimball (2006) construct a measure of technological change using structural estimation and find that it behaves very differently than the Solow residual. The introduction of goods market frictions by Diamond (1982) presents a promising avenue for demand shocks to influence total factor productivity. Bai, Rios-Rull, and Storesletten (2024), hereafter BRS, develop and estimate a two-sector neoclassical DSGE model with matching frictions. Output depends on firms' technology, inputs, and their efficiency in matching with customers. Increases in shopping effort, whether due to exogenous factors or as a response to other economic shocks, generate more matches, output, and measured total factor productivity. The disparity between potential output and matched output (value added) aligns with the essence of Keynes (1936) and reflects a reversal of causality in comparison to a standard TFP shock.

Our paper aims to quantify the importance of demand shocks on the Solow residual and other macroeconomic variables in an economy subject to goods market frictions. Our main contribution is leveraging the additional information provided by capacity utilization and sectoral data in our Bayesian estimation. Beyond conventional sectoral metrics (hours, consumption, investment), we incorporate total capacity utilization data from both nondurable and durable sectors. These two series exhibit strong comovement. As such, they provide a stringent test of the model in much the same way as the comovement of labor hours in the consumption and investment sectors. We find that the model estimated on sectoral data implies an essential role for search-based demand shocks. Moreover, capacity utilization plays a crucial role in estimation. Excluding them from the set of observables generates counterfactual negative comovement of these series.

We focus on capacity utilization because it provides direct information about the role of shopping effort in our model. The Federal Reserve Board constructs total capacity utilization as the ratio of an output index to capacity index for manufacturing, mining, and electric and gas utilities. This measure of capacity aims to quantify a plant's maximum sustainable output given its resource constraints. Following Qiu and Ríos-Rull (2022), we define sectoral capacity utilization within the model as the ratio of an output index to a capacity index–mirroring empirical measurements. In our environment with goods market frictions, we show that the growth rate of capacity utilization is a weighted sum of the growth rates of shopping effort and variable capital utilization.[2] Hence, leveraging capacity utilization data offers a much stricter test of the importance of search effort in relation to demand shocks.

Capacity utilization, in turn, is closely related to the Solow residual. We show that the growth rate of the Solow residual is the sum of growth rates of capacity utilization, technology, and mismeasurement of input shares.[3] Thus, total capacity utilization is a sufficient statistic for the contribution of demand shocks and variable capital utilization to the Solow residual. Furthermore, mismeasurement of input shares depends on sectoral movements in labor supply, providing additional justification for using this series as an observable variable.

Quantifying the role of demand shocks on the Solow residual depends crucially on key structural parameters pertaining to (1) the matching technology, (2) shopping disutility, and (3) stochastic processes of demand shocks. Denote the elasticity of the matching function as ϕ and the elasticity of disutility as η. Identification of these parameters is fundamental for measuring the transmission of demand shocks to the Solow residual.

Our identification strategy contrasts with Bai, Rios-Rull, and Storesletten (2024). BRS employ two datasets, one which features shopping time from the American Time Use Survey as a proxy for effort.[4] Additionally, they calibrate ϕ and η by making use of two nonlinear equations pertaining to the cross-sectional price dispersion for identical goods and the elasticity of shopping time with respect to expenditure. Shopping time is thus taken as a proxy for effort.

While leveraging identified micro moments to derive ϕ and η is generally compelling, using shopping time as a proxy for effort raises at least two concerns. First, as discussed by BRS, fluctuations in shopping effort should be construed more broadly to encompass changes in match efficiency, rather than solely focusing on time. Second, leisure activities can potentially contaminate shopping time. For instance, time spent browsing a store may reflect window shopping rather than genuine effort. An increased desire to find a particular item may lead

[2] The analytic expression for capacity utilization and it's relationship to search effort is found at equation (26) in Section 5.4. In general, few papers in the literature use capacity utilization, and some equate it with variable capital utilization (Born, Peter, and Pfeifer (2013), Christiano, Eichenbaum, and Trabandt (2016)).

[3] The analytic expression for the Solow residual and it's relationship to utilization is found at equation (27) in Section 5.4. The term related to mismeasurement of input shares results from misspecifying constant returns to scale in capital and labor and imposing perfectly competitive labor markets. This term would be absent if the econometrician knew the exact production technology.

[4] The other dataset uses the relative price of investment instead of shopping time. Both datasets include output, investment, and labor productivity.

to a shift towards actively searching for it and away from mere window shopping, resulting in an overall change in shopping time that reflects a combination of both factors.

Our model nests BRS as a special case, which we use as a laboratory for two purposes. First, we examine the alternative identification strategy using total capacity utilization data.[5] The exercise is in the spirit of Guerron-Quintana (2010), who investigates the choice of observable variables on estimated parameters in the context of a rich New Keynesian model. We drop shopping time as an observable and estimate ϕ and η directly using the dataset with the relative price of investment. We find that the posterior 90% probability band of ϕ ranges from 0.00 to 0.20, and the importance of shopping-disutility shocks in the variance decomposition drops significantly relative to BRS. Next, we estimate the same model but include capacity utilization as an observable series. Remarkably, the posterior probability band of ϕ changes to $(0.85, 0.90)$, and the contribution of demand shocks to the variance decomposition rises dramatically. Additionally, the standard deviation of capacity utilization rises ten-fold in this case compared to the former, similar to the empirical value. Second, we show that the estimated model generates sectoral comovement of labor and output, in line with the data, in contrast to a standard RBC model driven just by technology shocks.

Our general model includes several additional features to more accurately capture business cycle moments and the role of demand shocks. First, to account for movements in utilization more broadly, the model incorporates variable capital intensity subject to endogenous depreciation. Investment adjustment costs make the intensity of capital choice more valuable, and also helps reduce investment volatility and generate hump-shaped impulse responses. Second, external habits and limited factor mobility improve the autocorrelation and persistence properties of the model and, importantly, allow technology shocks to generate positive comovement of labor hours. We nevertheless find that search-based demand shocks play a very important role in the variance decomposition. Third, we disaggregate the consumption sector into nondurables and services using CES preferences. This step is necessary to construct

[5] Using total capacity utilization permits a better mapping between model and data relative to the quarterly measure of utilization developed by Fernald (2014): it can accommodate non-constant returns to scale, profits, and fixed costs. This is appealing because goods market frictions and competitive search generally require decreasing returns to scale, and fixed costs are a realistic ingredient linking output and productivity. Reassuringly, the two series behave similarly. If one defines Fernald utilization as the difference in cyclical components of total factor productivity and its utilization-adjusted counterpart, then it comoves closely with total capacity utilization.

utilization measures for durables and nondurables and map them into the data. Fourth, we incorporate fixed costs because they provide an alternative explanation of positive comovement between output and productivity, and also affect the relationship between intensity of capital use and total capacity utilization. Finally, the new stochastic processes, wage-markup shocks and investment-specific shopping disutility, help the model simultaneously fit sectoral data on hours and utilization as well as the relative price of investment.

The specification of the model is designed to not *a priori* favor demand or technology shocks. In addition to aiding in the fitting of second moments, the extra components provide technology shocks with greater flexibility to capture patterns of comovement. For instance, incorporating external habit formation and limited factor mobility into a standard RBC model allows a technology shock to align with labor comovement. This adjustment addresses the well-known sectoral comovement puzzle outlined by Christiano and Fitzgerald (1998). A similar logic applies to the influence of demand shocks on explaining capacity utilization and, consequently, the Solow residual. Omitting variable capital utilization, for instance, would skew the estimation toward search effort as the primary explanation for fluctuations in capacity utilization.

The general model yields the following insights. Search demand shocks account for nearly two-thirds of the forecast error variance of output and approximately 50% of the variance in the Solow residual. Moreover, these shocks significantly influence the relative price of investment and labor supply. The model effectively captures the comovement of consumption, investment, utilization series, and labor. Compared to the simpler BRS model, it provides a much better fit for the autocorrelation of sectoral labor, output, and utilization series. Collectively, this evidence underscores the crucial role that demand shocks play in explaining business cycles.

We use sectoral data for four reasons. First, sectoral comovement—the tendency for most sectors to move together—is a stylized fact and a central part of the official definition of the business cycle by the NBER. Christiano and Fitzgerald (1998) show that comovement holds across many fine-grained sectors, providing a stringent test which many business cycle models fail. Empirical capacity utilization measures also comove, expanding the set of comovement facts that discipline the model. Second, capacity utilization data relies on a measurable capacity index, which is unavailable for the services sector, requiring us to disaggregate consumption into nondurables and services. This allows utilization measures for durables and nondurables to align with observed data. Third, sectoral data is crucial for disciplining the model mechanism. In our simplified framework, the ratio of shopping effort between the consumption and investment sectors corresponds to the ratio of labor inputs. The use of sectoral data is thus informative about relative shopping effort which in turn affects sectoral capacity utilization and the Solow residual. In our general model, the shopping-effort ratio depends on the ratio of labor income and the ratio of marginal disutility of shopping across sectors. This more general specification allows the model to fit data on utilization, hours, and the relative price of investment jointly.[6] Fourth, the combination of sectoral labor and output data effectively captures labor productivity within each sector.

The set of observables we use for Bayesian estimation are demeaned growth rates of consumption, investment, labor hours in consumption, labor hours in investment, utilization in nondurable goods, utilization in durable goods, and the relative price of investment to consumption.[7] This set extends Katayama and Kim (2018) with the utilization measures but drops aggregate wages.

The stochastic processes encompass shocks to the trend in technology, stationary neutral technology, investment-specific technology, neutral shopping effort cost, investment-specific shopping effort cost, discount-factor, and wage markups. The latter capture unexpected spreads between the marginal product of labor and the wage rate paid by firms, and are a stand-in for shifts in labor market conditions and bargaining power. The model's components and shock structure build upon the framework introduced by BRS while integrating key elements from Schmitt-Grohé and Uribe (2012) and Katayama and Kim (2018).

Though our work aligns most closely with Bai, Rios-Rull, and Storesletten (2024), it is also greatly inspired by Michaillat and Saez (2015), who model and argue for a prominent role for aggregate demand on unemployment and idle time operating through goods market frictions. Similar to our approach, they regard rates of operation in the economy and their business cycle comovement as fundamental outcome variables in their own right. However, they do not formally discipline the model using time series data, relate goods market frictions

[6] Analytic expressions relating relative shopping effort across sectors to the sectoral allocation of labor is found at (11) and (21) in Section 4 and Section 5.2 respectively.

[7] Whereas BRS use labor productivity as an observable, the use of sectoral data on inputs and outputs means we effectively target labor productivity in each sector and also proxy for relative shopping effort.

to capacity utilization, or focus on sectoral comovement. Moreover, they model matching costs in terms of additional expenditures rather than effort. Appendix K carefully compares the two specifications and shows that it does not matter for the essence of the transmission mechanism but that it does affect the labor share of income, which is relevant for the Solow residual.

Section 2 provides key background facts on utilization and sectoral comovement. Section 3 lays out the model environment. Section 5 characterizes key equilibrium relationships. It also decomposes the growth rate of the Solow residual into structural forces and relates these to capacity utilization. Section 4 examines the informative role of capacity utilization in the nested BRS version of the model. Section 6 estimates the full model. It decomposes the forecast error variance decomposition and shows that crucial parameters related to goods market frictions and shocks are precisely estimated. Section 7 concludes. The appendices describe the data construction, derivation of equilibrium, calibration strategy, the identification of key parameters by estimating the model on artificial data, and the role of the matching costs. We sometimes omit time indices in describing static relationships to economize on notation.

2. Background and stylized facts on utilization and sectoral comovement

Total capacity utilization is provided by the Federal Reserve Board and encompasses 89 detailed industries (71 in manufacturing, 16 in mining, 2 in utilities).[8] These industries primarily correspond to the 3 or 4-digit North American Industry Classification System (NAICS) codes. Importantly for our purposes, estimates are available for durable and nondurable goods. In manufacturing, most capacity indices are based on responses to the Census Bureau's Quarterly Survey of Plant Capacity. The census is conducted quarterly at the establishment level. The probability that each establishment is selected is proportional to the value of shipments within each industry.

We decompose total capacity utilization into subcomponents for nondurables and durables. Figure 1 compares cyclical capacity utilization in durables and nondurables alongside real output and Fernald utilization, which we construct as the difference between cyclical TFP and the utilization-adjusted counterpart from Fernald (2014). The capacity utilization series

[8] This data can be downloaded at https://www.federalreserve.gov/datadownload/Choose.aspx?rel=G17.

series comove closely with each other and the Fernald measure and are procyclical, with total capacity utilization in durables exhibiting greater volatility.

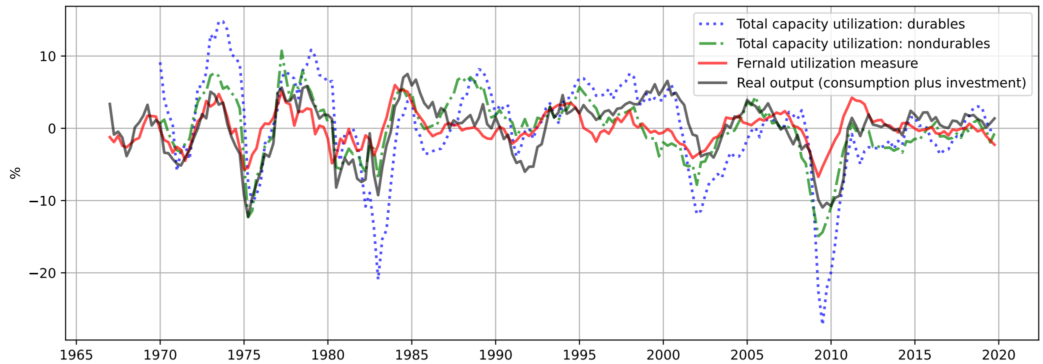

Figure 1: Total capacity utilization in non-durable and durable goods and output, here defined as consumption plus investment. Each underlying series is detrended via the Hamilton regression filter with the four most recent observations 8 quarters in the past ($p = 4, h = 8$).

Lastly, we examine business cycle statistics of the sectoral and utilization data. Table 1 presents the second moments of the series expressed in growth rates from 1964Q1-2019Q4. The use of growth rates aligns with the treatment of data in estimation, a standard practice since Smets and Wouters (2007), and eases comparison with other studies. The construction of hours uses the BLS Current Employment Statistics following Katayama and Kim (2018). The data appendix provides details, and Figure B.9 shows the detrended time series of hours in each sector alongside the aggregate measure. Following BRS, we define output as the sum of consumption and investment, consistent with our model framework. The findings indicate a strong correlation of 0.87 between labor hours, a moderate correlation of 0.54 between consumption and investment, and robust comovement between the utilization measures and investment, as well as labor hours in investment. Additionally, all series exhibit significant autocorrelation, except for labor productivity. Notably, investment, labor hours in investment, and utilization in durables display substantial volatility compared to consumption, labor hours in consumption, and utilization in nondurables.

	SD(x)	STD(x)/STD(Y)	Cor(x, I)	Cor(x, n_i)	Cor(x, x_{-1})
Y	0.87	1.00	0.94	0.70	0.47
C	0.44	0.51	0.54	0.44	0.48
I	2.14	2.46	1.00	0.73	0.41
n_c	0.57	0.66	0.66	0.87	0.67
n_i	1.94	2.23	0.73	1.00	0.64
Y/n	0.64	0.73	0.36	-0.28	0.10
p_i	0.51	0.58	-0.28	-0.22	0.44
$util_D$	2.27	2.61	0.69	0.84	0.55
$util_{ND}$	1.26	1.45	0.61	0.65	0.51

Table 1: Time range: $1964Q1 - 2019Q4$. Each underlying series is expressed in 100 quarterly log deviations. Here output is defined as the sum of consumption and investment. We use the symbols Y for output, C for consumption, I for investment, n_c for labor supply, in consumption, n_i for labor supply in investment, Y/n for labor productivity, p_i for the relative price of investment, and $util_D$ and $util_{ND}$ for the utilization of durables and nondurables, respectively. Appendix B describes the construction of variables in detail.

3. Model environment

3.1. Technology and markets

There is a unit mass of households and a unit mass of firms within each sector. There are three sectors: two for consumption (goods mc and services sc), and one for investment (i). Each sector j uses capital and labor to produce output. Moreover, capital can be used at a rate h, and production involves a fixed cost ν.[9] The economy grows with a stochastic trend X such that its growth rate $g_t = X_t/X_{t-1}$ is a stationary process with steady state $\overline{g}$. The production function now incorporates capital utilization and fixed costs:

$$F_j = z_j f(h_j k_j, n_j) - \nu_j X, \quad j \in \{mc, sc, i\} \tag{1}$$

$$f(hk, n) = (hk)^{\alpha_k} n^{\alpha_n} X^{1-\alpha_k} \tag{2}$$

Representation (1) and (2) ensures balanced growth, so that the share of fixed costs to output is stationary. Higher utilization of capital raises depreciation according to an increasing and

[9]By ‘fixed’ we mean that the cost does not vary with the choices of inputs. The cost scales with the stochastic trend X, so that the share of fixed costs to output is stationary on the balanced growth path.

convex function $\delta(\cdot)$. We assume the form

$$\delta^j(h) = \delta^K + \sigma_b(h-1) + \frac{\sigma_{aj}\sigma_b}{2}(h-1)^2, \quad j \in \{mc, sc, i\}, \sigma_{ac} \equiv \sigma_{amc} = \sigma_{asc}$$

where δ^K is an exogenous rate of depreciation. Note that $\delta^j(1) = \delta^K$, so that δ^K is the economy-wide steady-state depreciation rate of capital. Moreover, for each j, $\sigma_b = \delta^j_h(1)$ is the marginal cost of utilization in the steady state and $\sigma_{aj} = (1)\delta^j_{hh}(1)/\delta^j_h(1)$ is the elasticity of the marginal utilization cost with respect to rate h in the steady state. Alternatively, $1/\sigma_{aj}$ is the sector j elasticity of capital utilization with respect to the rental rate. We restrict the parameter σ_b to set steady-state capital use to unity in each sector. For parsimony, we also restrict the depreciation function to be the same within each subsector of consumption.

Investment is specific to each sector and features endogenous depreciation, as described above, and quadratic adjustment costs following Christiano, Eichenbaum, and Evans (2005).

$$k'_j = (1 - \delta_j(h_j))k_j + [1 - S(i_j/i_{j,-1})]i_j, \quad j \in \{mc, sc, i\}$$
$$S(x) = \frac{\Psi_K}{2}(x-1)^2$$

so that aggregate investment is $i = i_{mc} + i_{sc} + i_i$. We also use a common adjustment term Ψ_K for parsimony.[10]

Extending Moen (1997), there is a competitive search protocol in which each submarket is indexed by price, market tightness, and quantity (p, q, F). The measure of matches in each submarket is given by a sector-specific constant returns to scale matching function

$$M_j(D, T) = A_j D^{\phi} T^{1-\phi}, \quad 0 < \phi < 1, \quad j \in \{mc, sc, i\} \tag{3}$$

of aggregate shopping effort D and the measure of firms T. Market tightness is defined as search effort per firm location, $q = D/T$. We set $T = 1$, so that D measures market tightness. The probability that a unit of shopping effort is matched with a firm is $\Psi_{jd} = A_j D^{\phi-1}$ and the probability that a firm location is matched is $\Psi_{jT} = A_j D^{\phi}$. Once a match is formed, goods are traded at the posted price p_j. A household who exerts search effort d_j purchases

[10]We have also estimated the model with sector-specific investment adjustment cost functions and have not found significant differences in the results.

a real quantity of goods

$$y_j = d_j \Psi_{jd}(D) F_j, \quad j \in \{mc, sc, i\}$$

3.2. Households and firms

Households have preferences over search effort, consumption, and a labor composite following Bai, Rios-Rull, and Storesletten (2024). However, we also accommodate external habit formation, which is important to fit the data. Letting $\theta = (\theta_d, \theta_n, \theta_i)$ be a vector of preference shifters, household utility is given by

$$u(c, d, n^a, \theta) = \frac{\Gamma^{1-\sigma} - 1}{1 - \sigma} \tag{4}$$

where Γ is a composite parameter with external habit formation

$$\Gamma = c - haC_{-1} - \theta_d \frac{d^{1+1/\eta}}{1 + 1/\eta} - \theta_n \frac{(n^a)^{1+1/\zeta}}{1 + 1/\zeta}$$

where C is aggregate consumption and $d = d_{mc} + d_{sc} + \theta_i d_i$ is total search effort. Thus, θ_i is an exogenous wedge in the search cost of investment goods relative to consumption goods. The parameter η is the elasticity of shopping effort and ζ is the Frisch elasticity of labor supply.

Household consumption is a constant-elasticity-of-substitution aggregator of a bundle of goods y_{mc} and services y_{sc} with the associated price index:

$$\begin{aligned} c &= [\omega_{mc}^{1-\rho_c} y_{mc}^{\rho_c} + (1 - \omega_{sc})^{1-\rho_c} y_{sc}^{\rho_c}]^{1/\rho_c} \\ p_c &= \left(\omega_{mc} p_{mc}^{-\rho_c/(1-\rho_c)} + \omega_{sc} p_{sc}^{-\rho_c/(1-\rho_c)}\right)^{-\frac{1-\rho_c}{\rho_c}} \end{aligned} \tag{5}$$

such that $\omega_{mc} + \omega_{sc} = 1$ and the elasticity of substitution is given by $\xi = 1/(1 - \rho_c)$. Thus, p_{mc}/p_c and p_{sc}/p_c are the relative prices of nondurables and services to consumption overall.

Households have preferences with regard to the composition of labor they supply across sectors, following Horvath (2000) and Katayama and Kim (2018). Specifically, the labor composite n^a is

$$n^a = \left[\omega^{-\theta} n_c^{1+\theta} + (1 - \omega)^{-\theta} n_i^{1+\theta}\right]^{\frac{1}{1+\theta}} \tag{6}$$

where elasticity of substitution $1/\theta$ measures intersectoral labor mobility. The standard case of infinite marginal rate of substitution applies as $\theta \to 0$, in which case labor is perfectly mobile: $n^a \to n_c + n_i = n$.

A representative firm in sector $j \in \{mc, sc, i\}$ offers market bundle (p_j, D_j, F_j) and employs capital at rental rate R_j and labor at wage W_j in competitive spot markets to maximize profits. We introduce exogenous time-varying wage markups following the approach by Schmitt-Grohé and Uribe (2012) where a continuum of monopolistically competitive labor unions in each sector sell differentiated labor services.

Figure 2 summarizes the timing of the economy. First, aggregate shocks occur at the beginning of each period. Second, in each sector j, a firm posts a submarket offer (p_j, D_j, F_j). Third, given the submarket choice, households choose shopping, consumption, labor supply, and capital utilization. Firms simultaneously hire labor in a competitive spot market, which determines the wage. Fourth, matching takes place. Matched firms produce and sell. Fifth, the capital stock is updated in each sector, reflecting investment adjustment costs and endogenous depreciation.

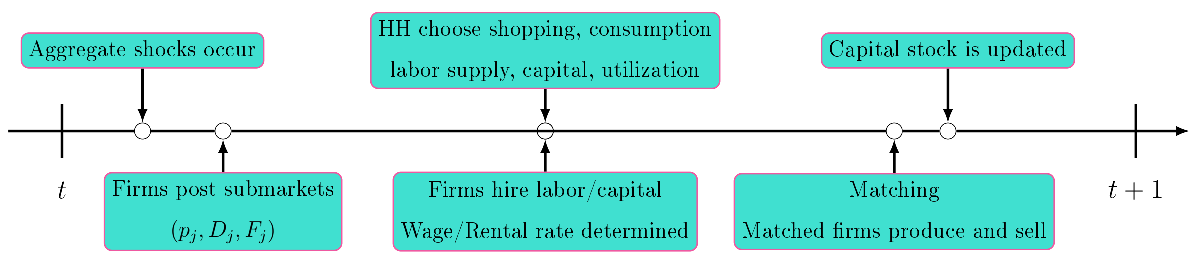

Figure 2: Timing

4. Demand shocks and the role of capacity utilization in a simple setting

We first highlight the productive role of demand and show that capacity utilization data can be used to discipline the key parameters underlying transmission. Consider the baseline model by Bai, Rios-Rull, and Storesletten (2024). This formulation is a special case of our general environment without habit formation ($ha = 0$); perfectly mobile labor ($\theta = 0$); a single consumption sector ($\rho_c \to 1$); no fixed costs in production ($\nu_j = 0$ for all j); fixed capital intensity ($\sigma_b \to \infty$); and no investment adjustment costs ($\Psi_K = 0$). In addition to demonstrating the importance of using capacity utilization data, we also show that sectoral comovement patterns, besides being important business cycle moments in their own right, inform the transmission of demand shocks in our environment.

To show how demand shocks can influence measured productivity, first consider a static version of BRS. The consumption good is produced using only labor ($\alpha_k \to 0$), so that (2) is simply $f(n) = n^{\alpha_n}$. A household who shops in submarket (p, D, F) chooses consumption, search effort, and labor supply in order to maximize their period utility:

$$\hat{V}(p, D, F) = \max_{d,c,n} u(c, d, n, \theta)$$
$$\text{s.t.} \quad c \leq d\Psi_d(D)F$$
$$pc \leq nW$$

Let $V = \max_{p,D,F} \hat{V}(p, D, F)$ be the value of the best submarket which firms must provide to households ensure participation. The value V is an equilibrium object but is taken as given by firms. A firm chooses which market bundle (p, D, F) to offer and the amount of labor n to employ to maximize period profits:

$$\max_{p,D,F,n} p\Psi_T(D)F - Wn$$
$$\text{s.t.} \quad \hat{V}(p, D, F) \geq V$$
$$zn^{\alpha_n} \geq F$$

Applying matching function (3), preferences (4), and aggregating shows that an equilibrium can be characterized as a tuple (C, D, W, n) satisfying optimal shopping, consistency of output, labor supply, and labor demand:

$$\theta_d D^{\frac{1}{\eta}} = \phi \frac{C}{D} \tag{7}$$
$$C = AD^{\phi} zn^{\alpha_n} \tag{8}$$
$$(1 - \phi)W = \alpha_n \frac{C}{n} \tag{9}$$
$$\theta_n N^{\frac{1}{\zeta}} = (1 - \phi)W \tag{10}$$

The GHH structure of preferences between consumption and shopping effort in (4) implies that the marginal rate of substitution is an increasing function of shopping effort: $-u_d/u_c = \theta_d d^{1/\eta}$. Equation (7) equates this marginal rate of substitution to the new matches induced by greater shopping effort—the product of $\partial M/\partial D = \phi\Psi_d$ and firm capacity F, which simplifies to $\phi C/D$. Equation (9) is a standard labor demand condition which equates the cost of labor to its value marginal product. Here the marginal product includes the probability of a firm finding a customer, $\Psi_T z f'(n) = z\alpha_n n^{\alpha_n - 1} AD^\phi$, so that labor demand is increasing in aggregate search effort. Equation (10) is a GHH labor supply condition: the marginal rate of substitution between consumption and labor, $-u_n/u_d = \theta_n n^{1/\zeta}$ equals the wage rate scaled by $(1-\phi)$. Moreover, the cost of labor is scaled by $(1-\phi)$. This feature arises from competitive search: increased output relaxes the household's participation constraint and thereby effectively lowers the input cost for the firm.

The labor share of income is $\tau \equiv Wn/C = \alpha_n/(1-\phi)$ using (9). Hence, the Solow residual is

$$SR \equiv C/n^\tau = AD^\phi z n^{\alpha_n - \tau} = AD^\phi z n^{-\alpha_n \phi/(1-\phi)}$$

Total factor productivity thus depends on technology, shopping effort, and mismeasurement of labor component. Capacity utilization is defined as the ratio of actual output (8) to capacity F

$$util \equiv C/F = AD^\phi$$

which measures how far realized output is from potential output. In the absence of any shocks to matching efficiency, the growth rate of capacity utilization is simply shopping effort scaled by the matching elasticity ϕ.

Figure 3 depicts the equilibrium using two graphs. The figure on the right shows the determination of search effort and consumption, for a given level of capacity F, as the intersection between (7) and (8). The figure on the left illustrates the determination of hours and wages, given consumption C, as the intersection between (9) and (10).

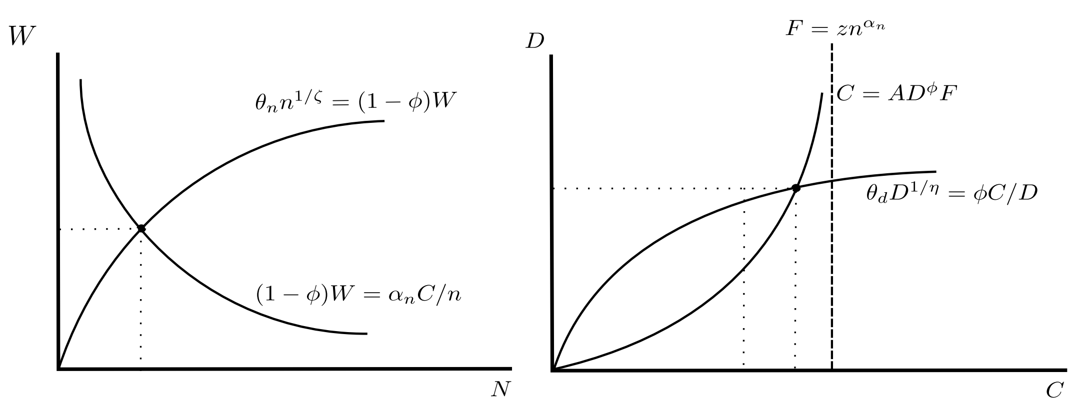

Figure 3: Equilibrium of static model

Now, let us consider a negative shock to the shopping disutility θ_d (Figure 4). The marginal cost of exerting shopping effort falls inducing households to shop more intensely, represented by the shopping curve shifting rightward. More shopping effort increases firms' matching rate and therefore boosts total production. This effect constitutes movement along the consumption curve from point 1 to point 2. To satisfy higher production levels, firms demand more workers, shifting the labor demand curve rightward and boosting labor hours and wages. Finally, more labor hours expands the productive capacity of firms, so the consumption curve shifts upward. This higher capacity further spurs shopping effort, represented by movement along the shopping curve from point 2 to point 3. The Solow residual therefore reflects both the initial increase in shopping effort from the demand shock followed by a further increase in shopping effort as households respond to increased capacity of firms. However, the rise of the Solow residual is slightly dampened by the mismeasurement of input shares. Notice that the demand shock to θ_d induces positive comovement across all variables in the economy and therefore resembles a standard technology shock to z.

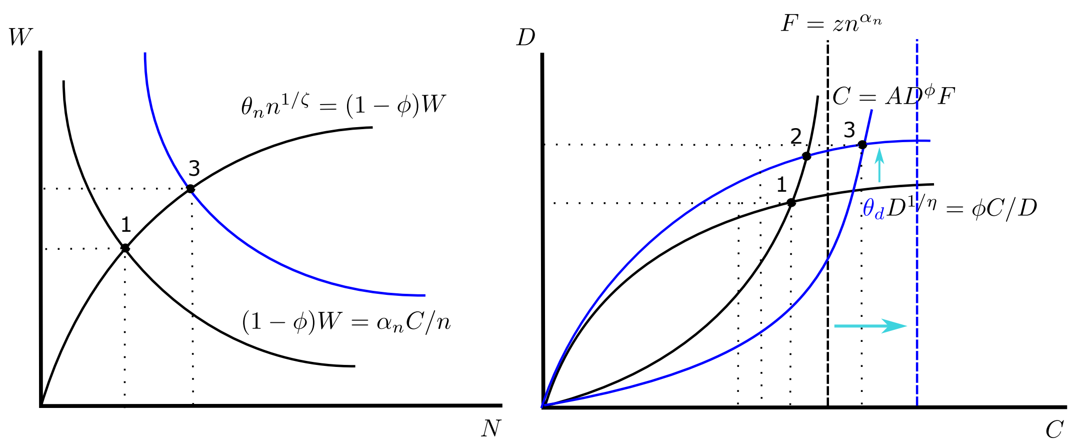

Figure 4: Reduction in shopping disutility in static model

Similarly, we examine the impact of a fall in labor disutility θ_n. This shocks shifts the labor supply curve rightward and increases capacity. The consumption curve shifts rightward and triggers a movement along the shopping curve, as before.[11]

Having described how demand shocks influence capacity, output, and the Solow residual in a static setting, we now consider the dynamic version of the simplified model, which includes

[11]In Appendix K, we also examine equilibrium in a static setting in which matching costs arise from expenditure à la Michaillat and Saez (2015). The causal effect of demand on output and productivity is essentially the same, but the labor share of income is α_n, and hence there is no input share mismeasurement in the Solow residual.

capital accumulation.[12] We estimate the shock processes $\{\theta_d, \theta_n, g, z, z_I\}$, each AR(1) with persistence $\{\rho_d, \rho_n, \rho_g, \rho_z, \rho_i\}$ and conditional standard deviation $\{\sigma_d, \sigma_n, \sigma_g, \sigma_z, \sigma_i\}$. This approach extends the set of shocks used by BRS to include neutral stationary technology shocks. While generally adhering to the same calibration strategy and targets, we now fix the following parameters: risk aversion $\beta = 0.99$, $\sigma = 2.0$, and Frisch elasticity $\zeta = 0.72$.[13] We estimate the model by adding total capacity utilization as an observable series to the BRS specification, which includes output, investment, labor productivity, and the relative price of investment. We then compare the estimates with and without capacity utilization.

Table 2 reports the prior distributions used for both specifications. In addition to ϕ and η, we specify distributions for the persistence parameters of nonstationary neutral technology, stationary neutral technology, investment-specific technology, labor supply, and shopping effort. We apply identical prior distributions for the conditional standard deviation and persistence of the stationary shocks. These conditional standard deviations follow an inverse gamma distribution with a mean of 0.01 and a standard deviation of 0.1. The persistence parameters have a prior mean of 0.6 and a standard deviation of 0.2.

Table 2: Prior distributions

Parameter	Distribution	Mean	Std
ϕ	Beta	0.32	0.20
η	Gamma	0.20	0.15
σ_{e_g}	Inv. Gamma	0.01	0.10
σ_x	Inv. Gamma	0.01	0.10
ρ_g	Beta	0.10	0.05
ρ_x	Beta	0.60	0.20

Table 2: Prior distributions. We use the symbol x as a shorthand for a shock in the set $\{\theta_d, \theta_n, z, z_I\}$.

Table 3 compares the posterior means and 90% probability bands of the key shopping-related parameters. In the first panel, the parameter ϕ is imprecisely estimated with a lower posterior mean. In fact, the 90% probability band includes essentially a null effect. By

[12] Appendix G lists the full set of equilibrium conditions.

[13] BRS also fix $\zeta = 0.72$ but they use $\sigma = 1$ and $\beta = 0.997$. We have also estimated the model with $\phi = 0.32$ and $\eta = 0.2$ as by BRS and obtained a similar variance decomposition as that paper.

contrast, when we add total capacity utilization, the posterior mean increases substantially to 0.88 and the estimate is precise. Estimates for the shopping cost elasticity η are also significantly higher and more precisely estimated. Generally, estimates of ρ_d and σ_d are more precise as well, though the properties differ. With the use of utilization data, demand shocks exhibit greater persistence, but their innovations become less volatile.

Table 3: Role of capacity utilization on parameter estimates

Parameter	BRS dataset		Add capacity utilization	
	Post. mean	90% HPD interval	Post. mean	90% HPD interval
ϕ	0.0978	[0.0001, 0.205]	0.883	[0.863, 0.906]
η	0.412	[0.282, 0.572]	1.87	[1.86, 1.90]
ρ_d	0.871	[0.775, 0.961]	0.928	[0.914, 0.941]
σ_d	0.0484	[0.0024, 0.0987]	0.0075	[0.0068, 0.0081]

Table 3: Estimation of baseline BRS model with to sets of observable series. The first considers growth rates of output, investment, labor productivity, and the relative price of investment. The second specification also considers total capacity utilization growth.

The top panel of Table 4 compares the standard deviations at the posterior mean of shocks θ_d, shopping effort D, and utilization $util$, where the last two are expressed in growth rates. The main result is that total capacity utilization is ten times more volatile even though shopping-effort shocks are less volatile and shopping effort has similar volatility. The key difference lies in the transmission of shopping effort to utilization through ϕ. The bottom panel highlights the role of these varying parameter estimates for the forecast error variance fraction attributable to demand shocks. It is very small in the former case but large in the latter, accounting for about two thirds of output, almost a third of labor productivity, and about half the Solow residual.

Table 4: Comparison of volatility and variance decomposition

	BRS dataset	Add capacity utilization
Volatility		
θ_d	9.84	2.00
D	1.54	1.69
$util$	0.15	1.49
FEVD		
Y	7.73	63.6
Y/N	2.49	27.0
SR	6.14	54.1

Table 4: The first sub-table documents standard deviations of shopping-related variables under two sets of observables. The second sub-table shows the fraction of the unconditional variance decomposition attributable to the demand shock θ_d. See Table 3.

These two exercises sharply illustrate the informative role of total capacity utilization. The shopping-related parameters more precisely estimated, demand shocks explain much more of the forecast error variance, and the volatility of total capacity utilization in the model rises ten-fold, much closer to the empirical value.[14]

Yet there are significant caveats to this analysis. First, in the absence of variable capital utilization, only shopping can influence total capacity utilization. Firms should also be able to select the intensity of capital use. To make the dynamic tradeoff more interesting and better fit investment data, there should also be investment adjustment costs. Then capacity utilization will reflect both shopping effort and intensity of capital use. Moreover, given the focus on productivity, it makes sense to incorporate fixed production costs. These empirically relevant costs help explain why productivity rises with output and also affect the contribution of capital intensity to capacity utilization.

Second, total capacity utilization is inappropriate as an economy-wide target since it is

[14] On page 28, footnote 14, BRS state that, in the absence of cross-sectional evidence, 'we find that the parameter ϕ is not well identified by the aggregate data. In particular, the resulting estimates of ϕ vary widely across data sets, ranging from 0.09 to 0.44 depending on whether we include or omit shopping time data.' By contrast, we find that ϕ *is* well-identified from aggregate data given the inclusion of capacity utilization.

only constructed for specific industries. In particular, it is not measured for consumption services, a large part of the economy. Enriching the model to include multiple sectors allows us to exploit dis-aggregated capacity utilization data in our estimation.

Third, the model struggles to fit aspects of sectoral data, which important for the transmission mechanism operating via goods market frictions. In the second specification, though the correlation of labor in each sector is not too far below the data (0.58), the autocorrelation is 0.18 for n_c and -0.01 for n_i. For consumption and investment, the respective autocorrelations are 0.28 and 0.20, well below the empirical values.

In the special case of BRS, relative shopping effort across sectors equals the relative labor allocation and the relative value of output:

$$\frac{D_c}{D_i} = \frac{n_c}{n_i} = \frac{C}{p_i I} \tag{11}$$

Equation (11) highlights the informative role of sectoral data: (1) the labor ratio pins down the ratio of shopping effort, (2) and labor inputs and sectoral output data provides information on sectoral labor productivity, which are in turn linked to the relative price of investment. Such data is especially relevant given our focus on a demand-based explanation of productivity.

Unfortunately, (11) raises the following challenge. The variables C, I, and p_i are observables in estimation and thus determine n_c/n_i. Trying to use n_c and n_i—or even just their ratio—as observables in estimation would induce stochastic singularity. The use of these series versus the relative price of investment becomes arbitrary.

Section 5 generalizes (11), breaking the one-for-one link between shopping effort and hours. The more general form arises from using sector-specific wage markup shocks, incorporating imperfect competition in the labor market in the vein of Schmitt-Grohé and Uribe (2012). Additionally, limited factor mobility facilitates sectoral comovement and dampens excessive volatility, and fixed costs permit a more general relationship between output and augment the contribution of capital intensity to total capacity utilization. We next analyze the general model featuring these additional frictions and shocks.

5. Equilibrium

5.1. Households

Let $(p, D, F) = \{(p_j, D_j, F_j) | j \in \{mc, sc, i\}\}$ be the set of submarkets available to a household. Let Λ be the aggregate state and let $\hat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p, D, F)$ be the value of the household conditional on these submarkets. Letting Φ be the set of available submarkets, then the value function is determined by the best combination of submarkets: $V(\Lambda, k_{mc}, k_{sc}, k_i) = \max_{\{p,D,F\}\in\Phi} \widehat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p, D, F)$. The household chooses search effort, labor hours, consumption, future capital, and utilization rates to solve:

$$\widehat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p, D, F) = \max_{d_j, n_c, n_i, y_j, i_j, k'_j, h'_j} u(y_{mc}, y_{sc}, d, n^a, \theta) + \beta\theta_b \mathbb{E}\{V(\Lambda', k'_{mc}, k'_{sc}, k'_i) | \Lambda\}$$

$$\text{s.t.} \quad y_j = d_j A_j D_j^{\phi-1} F_j, \quad j \in \{mc, sc, i\}$$

$$\sum_j y_j p_j = \pi + \sum_{j\in\{mc,sc,i\}} k_j h_j R_j + n_c W_c^* + n_i W_i^*$$

$$k'_j = (1 - \delta_j(h_j))k_j + [1 - S(i_j/i_{j,-1})]i_j, \quad j \in \{mc, sc, i\}$$

and the consumption and labor aggregators (5) and (6).

Appendix C derives each step of the household and firm problem. Here we focus on central and innovative features of equilibrium. The presence of a goods market friction leads households to optimally balance the marginal disutility of shopping with the marginal benefit of output in both the consumption and investment sectors:

$$-u_d = u_j \phi A_j D_j^{\phi-1} F_j \quad j \in \{mc, sc\} \tag{12}$$

$$-u_d \theta_i = \frac{u_{mc} p_i}{p_{mc}} \phi A_i D_i^{\phi-1} F_i \tag{13}$$

Equation (12) characterizes optimal shopping in each subsector of consumption. A summary statistic of the role of goods market frictions is the ratio of marginal utility and price multiplied by the marginal utility of wealth λ. It turns out that this wedge just depends on ϕ:

$$\frac{u_j}{\lambda p_j} = \frac{1}{1-\phi} \Rightarrow \frac{u_{mc}}{p_{mc}} = \frac{u_{sc}}{p_{sc}} \tag{14}$$

or $\phi = (u_j - \lambda p_j)/u_j$.

Recall from GHH preferences that $-u_d/u_j = \theta_d d^{1/\eta}$ is an increasing function of shopping effort alone. Combining this with equation (12), we conclude that households increase their shopping effort in response to higher firm capacity and matching probability, as well as a lower disutility of shopping effort. The condition for investment goods in equation (13) is similar, but with the marginal disutility adjusted by θ_i and the value of output computed in consumption units, accounting for the relative price.

Given (6), households optimally divide their labor hours between consumption and investment sectors:

$$\frac{n_c}{n_i} = \frac{\omega}{1-\omega}\left(\frac{W_c^*}{W_i^*}\right)^{1/\theta}$$

so that $1/\theta$ is the elasticity of substitution.

Taking the first order condition with respect to y_{mc} and y_{sc} and combining it with (5), we derive the demand curves for nondurables and services

$$y_j = p_j^{-\xi}\omega_j C \quad j \in \{mc, sc\} \tag{15}$$

where $\xi = 1/(1-\rho_c)$ repreents the elasticity of substitution. By using (15) together with (14), we find that $\lambda = \Gamma^{-\sigma}(1-\phi)$. Here, the term $\Gamma^{-\sigma}$ captures the direct influence from the marginal utility of consumption, while the goods market frictions introduce a wedge represented by ϕ.

Furthermore, the ratio of (12) and (13) provides insight into the relative price of investment:

$$\frac{p_i}{p_j} = \theta_i \frac{A_j}{A_i}\left(\frac{D_j}{D_i}\right)^{\phi-1} \frac{z_j}{z_i}\frac{f(h_j k_j, n_j) - \nu_j X}{f(h_i k_i, n_i) - \nu_i X} \tag{16}$$

If the price p_i increases compared to p_j, with capacity held constant, it implies that investment goods become more valuable in terms of consumption, leading to an increase in D_i/D_j. Additionally, equation (16) reflects the typical mechanism where an increase in investment capacity results in a decrease in the relative price p_i/p_j.

5.2. Firms and labor unions

A representative firm in sector $j \in \{mc, sc, i\}$ rents capital and hires labor in spot markets. We introduce exogenous time-varying wage markups following the approach by Schmitt-Grohé and Uribe (2012). A continuum of monopolistically competitive labor unions in sector j sell differentiated services, indexed by type s. The firm chooses inputs and market bundle (p_j, D_j, F_j) to maximize profits given the household participation constraint, technology, and differentiated labor. The problem is

$$\begin{aligned} \max_{k_j, n_j, p_j, D_j, F_j} \; & p_j A_j D_j^{\phi} F_j - \int_0^1 W_j(s) n_j(s) ds - R_j h_j k_j \\ \text{s.}t. \quad & \widehat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p_j, D_j, F_j) \geq V(\Lambda, k_{mc}, k_{sc}, k_i) \\ & z_j f(h_j k_j, n_j) - \nu_j X \geq F_j \\ & n_j = \left(\int_0^1 n_j(s)^{1/\mu_j} ds \right)^{\mu_j} \end{aligned}$$

The conditional demand for labor type s in sector j and corresponding wage index are

$$n_j(s) = \left(\frac{W_j(s)}{W_j} \right)^{-\frac{\mu_j}{\mu_j - 1}} n_j, \quad W_j = \left[\int_0^1 w_j(s)^{1/(1-\mu_j)} ds \right]^{1-\mu_j}$$

The labor union charges the firm a wage $W_j(s)$ and pays W_j^* to its members. It maximizes earnings subject to the conditional labor demand of the firm. The problem of the union is thus

$$\max_{W_j(s)} (W_j(s) - W_j^*) \left(\frac{W_j(s)}{W_j} \right)^{-\frac{\mu_j}{\mu_j - 1}} n_j \tag{17}$$

The solution to (17) is $W_j(s) = \mu_j W_j^*$. Within sector j, labor unions pay the same wage and firms choose identical quantities of labor within j: $W_j(s) = W_j, n_j(s) = n_j$ for all s. Labor unions provide additional earnings to households in the form of a wage rebate. Consequently, $W_j(s) - W_j^* = (\mu_j - 1) W_j^*$ represents a fixed component of the wage from the perspective of workers.[15]

[15] Labor unions here are a mechanism here designed entirely for the benefit of workers. Thus, the earnings rebated to the workers count as labor income, which matters for the mapping between model and data.

The factor demand curves for the firm are

$$(1-\phi)\frac{W_j}{p_j} = \alpha_n \frac{A_j D_j^{\phi} z_j f(h_j k_j, n_j)}{n_j} \quad j \in \{mc, sc, i\} \quad W_{mc} = W_{sc} \tag{18}$$

$$(1-\phi)\frac{R_j}{p_j} = \alpha_k \frac{A_j D_j^{\phi} z_j f(h_j k_j, n_j)}{h_j k_j} \quad j \in \{mc, sc, i\} \tag{19}$$

To provide an alternative characterization of the relative price of investment, we take the ratio of (18) for sectors i and $j \in \{mc, sc\}$:

$$\frac{p_i}{p_j} = \frac{n_i W_i}{n_j W_j} \frac{A_j}{A_i} \left(\frac{D_j}{D_i}\right)^{\phi} \frac{z_j f(h_j k_j, n_j)}{z_i f(h_i k_i, n_i)} \tag{20}$$

When D_j/D_i increases, while holding inputs and technology constant, it becomes easier to sell nondurables or services to customers, resulting in an increase in p_i/p_j. Equation (20) also takes into account the standard relationship where p_i/p_j decreases as investment-specific technology z_i/z_j rises.

Relationships (16) and (20) represent distinct curves that connect the relative price of investment p_i/p_j to relative shopping effort D_i/D_j. However, there is a slight complication in comparison, as fixed costs are present in (16) but not in (20). In the case of zero fixed costs, mutual consistency requires the following relationship:

$$\frac{D_i}{D_j} = \frac{1}{\theta_i} \frac{n_i W_i}{n_j W_j} \tag{21}$$

Relative shopping effort is determined by relative labor income and the variation in shopping disutility. Over the business cycle, the level of sectoral comovement influences n_i/n_j and thus provides information about relative shopping effort. However, compared with (11), in which the ratios of shopping effort and labor supply perfectly coincide, (21) is significantly more flexible. Limited factor mobility and wage markup shocks allow for additional fluctuation in relative wages, and the exogenous wedge θ_i also helps explain fluctuations in relative shopping effort.

The final three equilibrium conditions encompass Tobin's Q, optimal capital utilizations, and Euler equations pertaining to the selection of future capital. These conditions incorporate investment adjustment costs and depreciation resulting from capital utilization:

$$\frac{p_i}{1-\phi} = Q_j[1 - S'(x_j)x_j - S(x_j)] + \beta\theta_b\mathbb{E}\frac{\lambda'}{\lambda}Q'_jS'(x'_j)(x'_j)^2 \quad j \in \{mc, sc, i\}$$
$$\delta^j_h(h_j)Q_j = R_j \quad j \in \{mc, sc, i\}$$
$$Q_j = \beta\theta_b\mathbb{E}\frac{\lambda'}{\lambda}\left[(1-\delta^j(h'_j))Q'_j + R'_jh'_j\right] \quad j \in \{mc, sc, i\}$$

The variable Q_j represents the relative price of capital in sector j in terms of consumption. The presence of investment adjustment costs introduces a disparity between Q_j and $p_i/(1-\phi)$. Households determine the level of utilization such that the value of depreciated capital $\delta_h(h_j)Q_j$ is equal to the marginal product of capital R_j. Finally, households decide on the capital level that equates the marginal cost of foregone consumption Q_j to the anticipated discounted return. The expected return comprises the marginal product of capital in addition to the value of undepreciated capital, and the stochastic discount factor $\beta\theta_b\mathbb{E}\lambda'/\lambda$ transforms returns into current marginal utility.

5.3. Inducing stationarity

The specification of technology (1) implies that output, consumption, wages, and capital have the same stochastic trend as technology X_t, characterized by the growth rate $g_t = X_t/X_{t-1}$. The next section shows that the trend growth rate of the Solow residual is $g_t^{1-\tau}$ for labor share τ. Preferences regarding labor supply imply zero long-run wealth effects and hence ensure stationarity of labor supply. We adjust GHH preference weights to ensure stationarity of shopping effort. To focus on equilibrium fluctuations around stochastic trends, we divide each trending variable other than capital by the stochastic trend X_t. For the capital stock, we instead divide by X_{t-1} to maintain its predetermined nature.

5.4. The sector-specific Solow residual and capacity utilization

We construct the Solow residual for a specific sector in the model and relate it to capacity utilization and other structurally interesting components. Begin by expressing sectoral output as follows:

$$Y_{jt} = A_jD^{\phi}_{jt}(z_{jt}h^{\alpha_k}_{jt}X^{1-\alpha_k}_t k^{\alpha_k}_{jt}n^{\alpha_n}_{jt} - \nu_jX_t)$$

Let $\nu_j^R = \nu_j X/F_j$ be the fixed cost share of capacity. Then note that $\nu_j X/(z_j f(h_j k_j, n_j)) = \nu_j^R/(1+\nu_j^R)$, so that

$$Y_{jt} = \frac{A_j D_j^{\phi}(z_{jt} h_{jt}^{\alpha_k} X_t^{1-\alpha_k} k_{jt}^{\alpha_k} n_{jt}^{\alpha_n})}{1+\nu_{jt}^R}$$

Fernald (2014) constructs the sectoral Solow residual under the assumptions of constant returns to scale Cobb-Douglas technology in capital and labor, no fixed costs, and perfectly competitive factor markets. Accordingly, define the Solow residual in sector j as

$$SR_{jt} \equiv \frac{Y_{jt}}{k_{jt}^{1-\tau} n_{jt}^{\tau}} = \frac{A_j D_{jt}^{\phi}(z_{jt} h_{jt}^{\alpha_k} X_t^{1-\alpha_k} k_{jt}^{\alpha_k - 1+\tau} n_{jt}^{\alpha_n - \tau})}{1+\nu_{jt}^R} \tag{22}$$

where τ represents the steady-state labor income share. To express (22) in terms of growth rates, we introduce the symbol $dx_t = \Delta \log x_t$ and rewrite as

$$\begin{aligned} dSR_{jt} = \phi dD_{jt} + dz_{jt} + \alpha_k dh_{jt} + (1-\alpha_k)dX_t + (\alpha_k - 1 + \tau)dk_{jt} \\ +(\alpha_n - \tau)dn_{jt} - d(1+\nu_{jt}^R) \end{aligned} \tag{23}$$

From (23) we note that the trend net growth rate of the Solow residual is

$$(1-\alpha_k)dX_t + (\alpha_k - 1 + \tau)dX_t = \tau \log g_t$$

which implies that the Solow residual grows at the rate of output multiplied by the labor share of income. By introducing the log deviation $\tilde{\nu}_j^R = \log(\nu_j^R/\nu_{ss}^R)$, we can rewrite (23) as[16]:

$$dSR_{jt} = \phi dD_{jt} + dz_{jt} + \alpha_k dh_{jt} + (1-\alpha_k)dX_t + (\alpha_k - 1+\tau)dk_{jt} + (\alpha_n - \tau)dn_{jt} - \frac{\nu_{ss}^R}{1+\nu_{ss}^R}\Delta \tilde{\nu}_{jt}^R \tag{24}$$

Expression (24) decomposes the growth rate of the Solow residual into structural forces.

[16] Calculate

$$\log(1+\nu_j^R) \approx \log(1+\nu_{ss}^R) + \frac{1}{1+\nu_{ss}^R}(\nu_{jt}^R - \nu_{ss}^R) \approx \log(1+\nu_{ss}^R) + \frac{\nu_{ss}^R}{1+\nu_{ss}^R}\tilde{\nu}_{jt}^R$$

Hence, $d(1+\nu_{jt}^R) = \Delta \log(1+\nu_{jt}^R) \approx \frac{\nu_{ss}^R}{1+\nu_{ss}^R}\Delta \tilde{\nu}_{jt}^R$

It comprises a demand component ϕdD_{jt}, a capital utilization component $\alpha_k dh_{jt}$, a technology component $dz_{jt} + (1-\alpha_k)dX_t$, an input share mismeasurement component $(\alpha_k - 1 + \tau)dk_{jt} + (\alpha_n - \tau)dn_{jt}$, and a change in the fixed cost share component $[\nu_{ss}/(1+\nu^R_{ss})]\Delta\tilde{\nu}^R_{jt}$. The first component reflects the direct effect of goods market frictions, and there is also a general equilibrium feedback between higher shopping effort and the other components. Additionally, the calibration strategy establishes a relationship between the coefficients α_k and α_n in relation to ϕ. It is worth noting that the growth rate of cyclical labor productivity $d(Y_{jt}/n_{jt})$ has the same expression as (24), except that τ is replaced by 1. Therefore, $d(Y_{jt}/n_{jt}) = dSR_{jt} + (1-\tau)(dk_{jt} - dn_{jt})$. In general, we find that the Solow residual and labor productivity behave similarly in cyclical terms, and choose to emphasize the former because of its significance in the literature.

We next turn to capacity utilization and relate it to the Solow residual. Following Qiu and Ríos-Rull (2022), we define capacity in sector j as

$$cap_j = z_j k_j^{\alpha_k} n_j^{\alpha_n} X^{1-\alpha_k} - \nu_j X$$

Consistent with the definition from the Federal Reserve Board, capacity utilization in sector j is the ratio of output to capacity:

$$\begin{aligned} util_{jt} \equiv \frac{Y_{jt}}{cap_{jt}} &= \frac{A_j D^{\phi}_{jt}(z_{jt} h^{\alpha_k}_{jt} X^{1-\alpha_k}_t k^{\alpha_k}_{jt} n^{\alpha_n}_{jt} - \nu_{jt} X_t)}{z_{jt} k^{\alpha_k}_{jt} n^{\alpha_n}_{jt} X^{1-\alpha_k}_t - \nu_{jt} X_t} \\ &= \frac{A_j D^{\phi}_{jt}(z_{jt} h^{\alpha_k}_{jt} (k_{jt}/X_t)^{\alpha_k} n^{\alpha_n}_{jt} - \nu_{jt})}{z_{jt} (k_{jt}/X_t)^{\alpha_k} n^{\alpha_n}_{jt} - \nu_{jt}} \end{aligned} \tag{25}$$

Capacity utilization is stationary since k_j grows at the same rate g as X on the balanced growth path. Expressing (25) in growth rates yields

$$dutil_{jt} = \phi dD_{jt} + (1+\nu^R_{ss})\alpha_k dh_{jt} \tag{26}$$

The growth rate of utilization equals that of shopping effort scaled by ϕ and capital utilization scaled by $(1+\nu^R_{ss})\alpha_k$. Therefore, higher fixed costs amplify the relative weight of capital utilization to shopping effort.

By comparing (26) and (24), we see that shopping effort enters with the same weight ϕ

but that the weight of capital utilization differs due to the presence of fixed costs. In the special case of zero fixed costs, the Solow residual growth rate simplifies into the sum of growth rates of utilization, technology, and mismeasurement of input shares.

$$dSR_{jt}|_{\nu_j=0} = dutil_{jt} + dz_{jt} + (1-\alpha_k)dX_t + (\alpha_k - 1 + \tau)dk_{jt} + (\alpha_n - \tau)dn_{jt} \qquad (27)$$

Our definition of the sectoral Solow residual follows the methodology outlined by Fernald (2014). This approach mitigates potential additional composition bias that may arise from employing an aggregate production technology. Furthermore, it aligns sensibly with the concept of utilization, which is only applicable to specific industries. Accordingly, the aggregate Solow residual and capacity utilization can be defined as the output-weighted average of sectoral values, as consistent with Fernald (2014):

$$SR = \sum_j \frac{Y_j}{Y} SR_j, \quad util = \sum_j \frac{Y_j}{Y} util_j$$

To a first-order approximation, the linearized expressions (24), (26), and (27) also apply to their respective aggregates. This allows us to quantify the proportion of Solow residual variance explained by the utilization component, $Var(dutil)/Var(dSR)$.

We have discussed the Solow residual and capacity utilization in terms of growth rates to facilitate comparison with empirical practice (e.g., Fernald (2014)) and to maintain consistency with the form of variables used in the observation equations and for business cycle statistics. In Appendix E, we provide a similar comparison between the cyclical deviations of the Solow residual and capacity utilization.

6. Main quantitative analysis

6.1. Stochastic processes

The growth rate of the stochastic trend $g_t = X_t/X_{t-1}$ follows an AR(1) process in logs, as Bai, Rios-Rull, and Storesletten (2024):

$$\log g_t = (1-\rho_g)\log \overline{g} + \rho_g \log g_{t-1} + e_{g,t}$$

where $e_{g,t} \sim N(0, \sigma_g^0)$. Here, $\log X_t$ follows a random walk with drift.

We also consider a stationary neutral shock z_c and an investment-specific shock z_i. We let $z_i \equiv z_c z_I$ where z_I is independent of z_c. Finally, there are disturbances to general shopping disutility θ_b, investment-specific shopping disutility θ_i, the discount factor θ_d, labor supply θ_n, and wage markups μ_c and μ_i. We do not include consumption preference shocks because they can be replicated by sequences of labor supply, shopping-disutility, and discount-factor shocks.

Each stationary shock in the set $v = \{\theta_b, \theta_d, \theta_n, \theta_i, z_c, z_I, \mu_c, \mu_i\}$ follows an AR(1) process

$$\log v_t = \rho_v \log v_{t-1} + e_{v,t}$$

where $e^0_{v,t} \sim N(0, \sigma^0_v)$.

6.2. Bayesian estimation

The Bayesian framework allows us to incorporate prior (e.g.) microeconomic evidence, quantify parameter uncertainty, decompose the forecast error variance of each shock, and compare the fit of models via the marginal likelihood. Another appealing feature is that the marginal likelihood also implicitly penalizes parameter complexity. If the expansion of the parameter space is irrelevant for fitting the data, then this reduces the prior probability mass of parameters that do help fit the data and thereby lowers the marginal likelihood.

Along these lines, we estimate the general model using Bayesian techniques with quarterly data from $1964Q1$ to $2019Q4$. The likelihood of the data sample Y given the estimated parameters Θ is denoted as $L(Y|\Theta)$. By incorporating the prior parameter distribution $P(\Theta)$, the posterior density is proportional to $L(Y|\Theta)P(\Theta)$. We employ the random walk Metropolis Hastings algorithm, which is a standard practice for drawing from the posterior distribution of Θ. We use the following observables expressed in growth rates: consumption C, investment I, labor hours n_c and n_i, sectoral utilization $util_{ND}$ and $util_D$, and the relative price of investment p_i. This dataset is similar to Katayama and Kim (2018), but we include the utilization variables and exclude wages. Formally, the vector of observables $\mathbf{Y}_t$ is

$$\mathbf{Y}_t = \begin{bmatrix} dC_t & dI_t & dn_{ct} & dn_i & dutil_{ND,t} & dutil_{D,t} & dp_{it} \end{bmatrix}'$$

The vector of estimated parameters Θ consists of the persistence and conditional standard deviations for shocks, the risk aversion parameter σ, the habit formation parameter ha, the parameter ζ closely related to the Frisch elasticity of labor supply, the fixed cost share parameter of potential output ν^R, the elasticity of depreciation with respect to capital utilization σ_{ac} and σ_{ai}, the investment adjustment cost parameters Ψ_K, the inverse of the intersectoral elasticity of labor supply θ, and the elasticity of substitution between nondurables and services ξ. We focus most on the elasticity of the matching function with respect to shopping effort ϕ and the elasticity of shopping cost η.

To calibrate the remaining parameters, we use long-run targets, normalizations, and a subset Θ_R of the estimated parameters. Table 5 presents the results. The fixed exogenous parameters include the discount factor β, average growth rate $\overline{g}$, gross wage markup μ, the share ω of labor hours in consumption, and the share of services in consumption. Following the approach of Katayama and Kim (2018) and standard practice, we set $\beta = 0.99$, $\overline{g} = 0.45\%$, $\mu = 1.15$, and $\omega = 0.8$. We pin down the weight of services ω_{sc} in the consumption aggregator as the average share of services in consumption, $\omega_{sc} = p_{sc}y_{sc}/C = 0.65$ over the sample.

The second set of parameters Θ_R is estimated and used to calibrate other parameters. These are the parameters of risk aversion σ, labor supply ζ, elasticity of the matching function ϕ, elasticity of shopping effort cost η, fixed cost share ν_R, and habit persistence ha.

The third set of parameters determines the choice of units but does not impact the cyclical behavior of the economy. We normalize output and the relative price of services and investment to unity, effectively determining the level parameters of technology for each sector. Additionally, we set the fraction of time allocated to work as 30%, which, in conjunction with other parameters, specifies the value of θ_n. To achieve a target capacity utilization of 81% in each sector, we adjust the level parameters A_j of the matching function accordingly. Finally, by setting the capital utilization rate to 1, we obtain the value for σ_b.

The fourth set of parameters are determined through long-run targets and the estimated parameters in the second group. The long-run targets includes those chosen by Bai, Rios-Rull, and Storesletten (2024). These are an investment-share of output of 20%, an annual capital-to-output ratio of 2.75, and a labor share of income of 67%. These in turn pin down the parameters δ, α_k and α_n. Appendix D discusses the calibration in detail. Note that, at the posterior mean, $\phi = 0.75$, $\nu^R = 0.24$, and $\alpha_k = 0.28$. Hence, from (26), $dutil_t \approx 0.75dD_t + 0.35dh_t$.

Targets	Value	Parameter	Calibrated value/posterior mean
First group: parameters set exogenously			
Discount factor	0.99	β	0.99
Average growth rate	1.8%	$\overline{g}$	0.45%
Gross wage markup	1.15	μ	1.15
Labor share in consumption	0.8	ω	0.8
Share of services in consumption	0.65	ω_{sc}	0.65
Second group: estimated parameters used for calibration			
Risk aversion	–	σ	1.71
Frisch elasticity	–	ζ	0.93
Elasticity of matching function	–	ϕ	0.75
Elasticity of shopping effort cost	–	η	0.34
Fixed cost share of capacity	–	ν_R	0.24
Habit persistence	–	ha	0.63
Third group: normalizations			
SS output	1	z_{mc}	0.44
Relative price of services	1	z_{sc}	0.63
Relative price of investment	1	z_i	0.37
Fraction time spent working	0.30	θ_n	1.8
Capacity utilization of nondurables	0.81	A_{mc}	2.2
Capacity utilization of services	0.81	A_{sc}	1.4
Capacity utilization of investment sector	0.81	A_i	2.9
Capital utilization rate	1	σ_b	0.033
Fourth group: standard targets			
Investment share of output	0.20	δ	1.4%
Physical capital to output ratio	2.75	α_k	0.28
Labor share of income	0.67	α_n	0.13

Table 5: Calibration targets and parameter values. Here we calibrate a subset of parameters using long-run targets and the posterior mean of the estimated parameters $\sigma, \zeta, \phi, \eta, \nu_R$ and ha.

Table 6 presents the posterior estimates along with the prior distributions. Of particular interest is the posterior mean of the matching function elasticity ϕ, which is estimated to be 0.752. This suggests that the search-based demand channel plays a significant role in the model. The posterior mean values of σ (1.71) and ha (0.63) are consistent with previous findings in the literature.

The inverse of the elasticity of substitution of labor, θ, has a posterior mean of 1.06, substantially less than the value 2.57 estimated by Katayama and Kim (2018). This difference can be attributed to the use of search demand shocks and absence of wealth effects, which naturally induce complementarity. The elasticity of substitution ξ between nondurables and services has a posterior mean of 0.865, which is fairly close to the prior mean, and is somewhat more concentrated compared to the prior distribution. The fixed cost share ν_R has a posterior mean of 0.244, somewhat higher than the prior mean. Relative to Qiu and Ríos-Rull (2022), we need a somewhat higher fixed cost share to fit the disaggregated data.[17]

We estimate a high posterior mean of 8.36 for the investment adjustment cost parameter Ψ_K. Intuitively, high investment adjustment costs are necessary to permit a high volatility of utilization without triggering excessively high volatility of investment. The estimated elasticities of the marginal cost of capital utilization are higher for consumption than for investment, which aligns with the greater volatility of investment and capacity utilization in durable goods. However, the estimated values are lower than those reported in Katayama and Kim (2018), in order to fit the volatility of the utilization series.

We estimate generally high values for the persistence parameters. This is especially the case for the shopping-effort shocks, with posterior means of 0.925 and 0.985, respectively. The mean persistence of the neutral shopping-effort is very close the value of 0.928 obtained in Section 4 and BRS's own estimate in Table 3. The posterior mean of ρ_g is 0.483, which indicative of moderate peristence of shocks to the stochastic trend, is moderately lower than the value 0.602 reported by BRS in Table 3. We also find greater persistence of wage markup shocks in investment (0.977) compared to consumption 0.804, a feature which seems necessary to fit the utilization data in conjunction with hours and the relative price of investment. The investment wage markup shock also has a far greater conditional standard deviation.

[17] Abraham, Bormans, Konings, and Roeger (2021) estimate the fixed cost share of output using Belgian firm-level panel data at 23.4%.

Appendix J assesses the identifiability of these parameters by estimating the model on artificial data generated from the model evaluated at the posterior mean. Most parameters, and in particular ϕ, η and the stochastic processes for search-based demand shocks, are well-identified.

Table 6: Bayesian estimation of baseline model

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	5%	HPD 95%
σ	beta	1.500	0.2500	1.712	0.3418	1.2241	2.2344
ha	beta	0.500	0.2000	0.634	0.0726	0.5097	0.7408
ζ	gamm	0.720	0.2500	0.925	0.1616	0.6476	1.1451
ϕ	beta	0.320	0.2000	0.752	0.1079	0.5788	0.9355
η	gamm	0.200	0.1500	0.344	0.1100	0.2018	0.5231
ξ	gamm	0.850	0.1000	0.865	0.0711	0.7595	0.9885
ν_R	beta	0.200	0.1000	0.244	0.1118	0.0913	0.4329
σ_{ac}	invg	1.000	1.0000	1.897	0.3646	1.2892	2.5271
σ_{ai}	invg	1.000	1.0000	0.444	0.0910	0.2980	0.5890
Ψ_K	gamm	4.000	1.0000	8.364	2.1355	4.3518	11.2623
θ	gamm	1.000	0.5000	1.059	0.3647	0.4592	1.6688
ρ_g	beta	0.100	0.0500	0.508	0.0892	0.3695	0.6572
ρ_z	beta	0.600	0.2000	0.743	0.0520	0.6567	0.8238
ρ_{zI}	beta	0.600	0.2000	0.865	0.0355	0.8058	0.9222
ρ_n	beta	0.600	0.2000	0.987	0.0078	0.9764	0.9998
ρ_d	beta	0.600	0.2000	0.924	0.0221	0.8891	0.9618
ρ_{dI}	beta	0.600	0.2000	0.984	0.0100	0.9695	0.9993
ρ_b	beta	0.600	0.2000	0.928	0.0221	0.8928	0.9644
$\rho_{\mu c}$	beta	0.600	0.2000	0.791	0.1954	0.4813	0.9977
$\rho_{\mu i}$	beta	0.600	0.2000	0.980	0.0223	0.9474	1.0000
e_g	gamm	0.010	0.0100	0.006	0.0015	0.0039	0.0083
e_z	gamm	0.010	0.0100	0.008	0.0011	0.0058	0.0094
e_{zI}	gamm	0.010	0.0100	0.015	0.0024	0.0107	0.0185
e_n	gamm	0.010	0.0100	0.008	0.0016	0.0060	0.0110
e_d	gamm	0.010	0.0100	0.099	0.0180	0.0708	0.1285
e_{di}	gamm	0.010	0.0100	0.017	0.0020	0.0142	0.0205
e_b	gamm	0.010	0.0100	0.009	0.0054	0.0015	0.0171
$e_{\mu c}$	gamm	0.010	0.0100	0.001	0.0005	0.0001	0.0013
$e_{\mu i}$	gamm	0.010	0.0100	0.022	0.0046	0.0149	0.0299

Table 6: Prior and posterior distribution.

Table 7 documents the unconditional forecast error variance decomposition of the model. Technology shocks and shopping-effort shocks are the primary drivers of forecast error variance in output, the Solow residual, investment, the relative price of investment, and variable capital utilization. Shopping-effort shocks have a particularly significant impact on utilization. The only significant contribution of discount-factor, wage markup, and labor supply shocks lies in explaining portions of labor in consumption and investment. However, the fraction of consumption-sector labor explained by labor supply shocks (35.2%) is second only to shopping-effort shocks.

Here our primary focus is on the Solow residual and utilization. Shopping-effort and technology shocks play similarly important roles for the former, but the search demand shocks explain over 70% of utilization. Hence, the evidence strongly supports a powerful causal channel of demand shocks into productivity.

Table 7: Unconditional forecast error variance decomposition

	Technology	Labor Supply	Shopping Effort	Discount Factor	Wage Markup
Y	27.78	0.10	71.06	1.01	0.05
SR	42.27	4.29	51.63	0.86	0.95
I	31.31	0.08	63.18	5.41	0.02
p_i	62.39	0.02	36.86	0.30	0.43
n_c	5.71	35.20	53.64	5.12	0.33
n_i	21.10	3.43	55.26	3.44	16.76
$util$	27.15	0.11	71.82	0.90	0.03
D	0.42	0.00	99.56	0.01	0.00
h	22.18	0.08	77.30	0.43	0.01

Table 7: Unconditional forecast error variance decomposition for variables in growth rates. Shocks are grouped in respective categories.

Table 8 compares the log marginal likelihood, posterior mean of ϕ, variance decomposition, and second moments for various specifications of the model. We calculate the log marginal data density using the modified harmonic mean estimator. The baseline model accounts for two thirds of the variance decomposition of output and nearly half of the Solow residual. The relative variance of utilization to the Solow residual is 0.87. These statistics are similar in the absence of variable capital utilization but fall somewhat without fixed costs. This suggests significant complementarity between the demand channel and fixed costs.

Table 8: Comparison of model specification

					Remove			
	Data	Baseline	Perfect labor mobility	Common wage markup	Fixed cost	VCU	SDS	SDS and utilization data
LML	–	4556.7	4529.3	2923.7	4566.8	4473.8	2564.9	–
Δ LML	–	0	-27.4	-1633	10.1	-82.9	-1991.8	–
Posterior mean ϕ	–	0.75	0.39	0.94	0.94	0.36	0.72	0.52
FEVD(Y, SDS)	–	71.06	62.61	5.39	71.16	69.25	–	–
FEVD(SR, SDS)	–	51.63	49.49	4.02	46.76	57.87	–	–
Var$(util)$/Var(SR)	–	1.4	0.72	0.32	2.02	0.74	2.21	0.19
std(Y)	0.87	1.6	2.02	7.57	1.38	2.21	207.71	0.64
std$(util_{ND})$	1.26	1.24	1.18	5.08	1.21	1.55	161.65	0.35
std$(util_D)$	2.27	3.2	2.69	12.81	3.62	2.43	266.65	1.14
std(n_c)	0.57	0.69	0.67	2.92	0.67	0.89	71.31	0.56
std(n_i)	1.94	2.47	2.77	12.25	2.26	2.01	344.8	1.87
Cor(C, I)	0.54	0.64	0.74	0.09	0.52	0.57	0.999	0.24
Cor$(util_{ND}, util_D)$	0.75	0.45	0.76	-0.27	0.29	0.63	0.999	-0.6
Cor(n_c, n_i)	0.87	0.59	0.40	-0.92	0.66	0.27	0.986	0.83
Cor$(util_{ND}, util_{ND,,-1})$	0.51	0.31	0.25	0.46	0.23	-0.05	0.999	0.27
Cor$(util_D, util_{D,-1})$	0.55	0.48	0.47	0.37	0.48	0.25	0.999	0.26

Table 8: Comparison of log marginal likelihood, posterior mean of ϕ, variance decomposition, and second moments for various specifications of the model. The log marginal likelihood (LML) is calculated using the modified harmonic mean. The first column describes relevant empirical moments, and the second column corresponds to the baseline model. The third and fourth columns present estimates of the model with perfect labor mobility ($\theta = 0$) and only a common wage markup shock, respectively. The fifth and sixth columns present estimates in which fixed costs and variable capital utilization are removed. The seventh column removes search-based demand shocks, and the eighth column also removes the utilization series from the set of observables.

We next examine second moments, delving more specifically into the model's ability to fit the data. The baseline model tends to overestimate the volatility of output but fits the volatility of the utilization series and labor hours quite well. It captures the correlation between consumption and investment. well as the correlations of the utilization series well and that of labor hours moderately well. It also does a reasonable, but not excellent, fit of the correlation of utilization and labor hours. Finally, the model fits the autocorrelation of the utilization series reasonably well, matching that of durables and coming close to the autocorrelation of nondurables.

The third column shows the results after estimating the model with perfect labor mobility ($\theta = 0$). Even though the posterior mean of ϕ decreases to 0.39, search-based demand shocks continue to have an outsized role in the variance decomposition. The model fits data worse overall but better captures the correlation of the utilization variables. The fourth column re-estimates the model under common wage-markup shocks. As expected from our discussion of the shopping ratio (21), this omission dramatically worsens model fit. The reason is that the shopping-effort ratio is now much more directly tied to the labor ratio, and it loses flexibility in fitting the comovement of utilization. Consequently, at the posterior mean, the correlations of utilization (-0.27) and especially labor hours (-0.92) become negative. The volatilities are dramatically higher as well. The substantial reduction in model fit is reflected in a 1,633 reduction in the log marginal likelihood compared to the baseline.

In the next two columns, we remove fixed costs and variable capital utilization, one-by-one. Both of these ingredients can be considered important robustness checks on the search-based demand channel. Removing fixed costly does not greatly impact model fit. It slightly improves the comovement of labor hours and attenuates the excess volatility of output. However, it generates too low of a correlation of the utilization variables (0.29) and further reduces the autocorrelation of the utilization of nondurables. Removing variable capital utilization, however, is far more detrimental. The log marginal likelihood falls by 82.9. Intuitively, the model loses flexibility in explaining utilization and output. There is more excess volatility of output in this case, and the implied autocorrelation of the utilization variables collapses, becoming slightly negative for nondurables.

The penultimate (seventh) column removes search-based demand shock. This specification resembles Katayama and Kim (2018), but goods market frictions still operate through other shocks. It is immediately evident that this change completely prevents the model from fitting the data: the log marginal data density collapses by nearly $1,992$, the standard deviations exceed those of the data by two orders of magnitude, and the correlations and autocorrelations are nearly unity. Intuitively, the capacity utilization data roughly pins down the sectoral shopping efforts, and the model lacks freedom to fit sectoral labor and output and the relative price of investment jointly. The appendix shows that the special case of a unitary consumption sector, no fixed costs, and no investment adjustment costs gives rise to stochastic singularity.

The final column also removes utilization data, making the set of observables similar to Katayama and Kim (2018). Estimating this specification confirms that the model can fit non-utilization data reasonably well. The volatility of output (0.64) and labor hours $(0.56, 1.87)$ are close to the empirical values. The model also fits the labor comovement well (0.83), thought the comovement of consumption and investment is too low (0.24). However, the volatility of the utilization variables is too low, and their comovement is sharply negative (-0.6). That is, absent search-based demand shocks, the model fits standard macro series well at the expense of matching the volatility and comovement of utilization. A corollary is that a multisector real business cycle model without the goods market frictions would face the same problem.

To better interpret these results, we examine impulse responses of consumption, investment, their respective labor inputs, utilization variables, and the relative price of investment from the baseline model with the parameters set to the posterior mean. For ease of comparison, we present the impulse responses in growth rates. The utilization variables consist of the observable subcomponents, durables and nondurables, together with aggregate utilization. A large fraction of aggregate utilization reflects services and is thus unobservable. We also include shopping effort D and capital intensity h in reference to Equation (26): $dutil_t \approx 0.75 dD_t + 0.35 dh_t$.

Figure 5 plots the impulse response to a unit standard deviation reduction in shopping effort. This shock prompts households to increase their shopping effort, leading to a boost in matching and utilization. As a result, firms experience a higher demand for labor in both sectors, thanks to their improved ability to match. Consequently, the shock generates positive comovement in the growth rates of sectoral output, sectoral input, and utilization in the nondurables and durables sectors. As expected, the Solow residual rises on impact. Moreover, the relative price of investment is countercyclical as in the data.

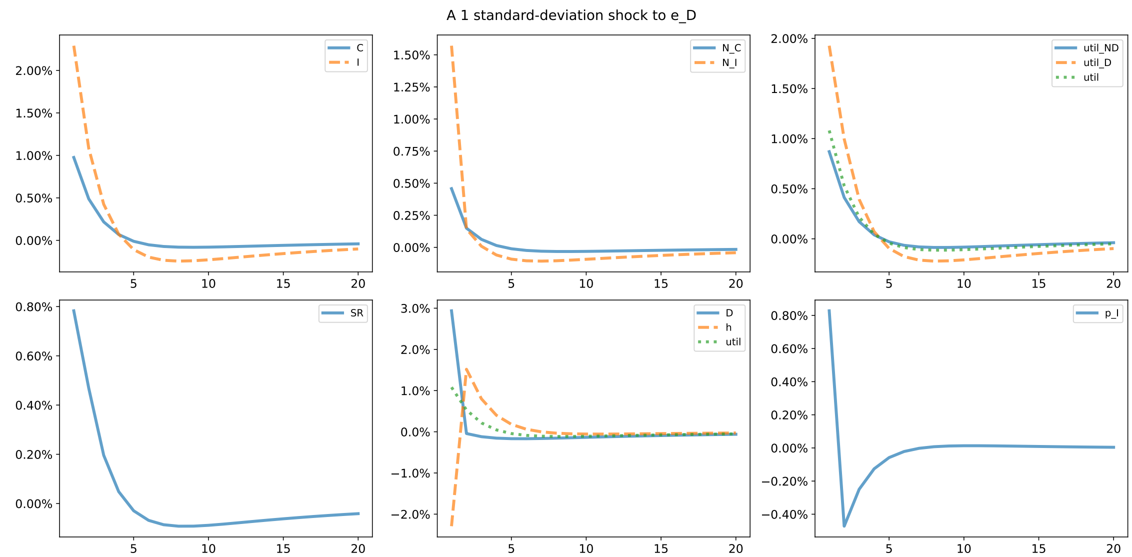

Figure 5: A unit standard deviation negative shock e_d to shopping effort in baseline model with parameters set at the posterior mean. The outcome variables are presented in growth rates.

Figure 6 plots the impulse response to a unit standard deviation discount-factor shock. Households are more patient, which raises the desire to consume in the future relative to the present. As a result, consumption falls while investment rises. Additionally, there is an increase in utilization in the durables sector but a decrease in utilization in the nondurables sector. Limited factor mobility attenuates but does not prevent the fall in labor in the consumption sector. Contrary to the data, there is positive comovement of investment and its relative price.

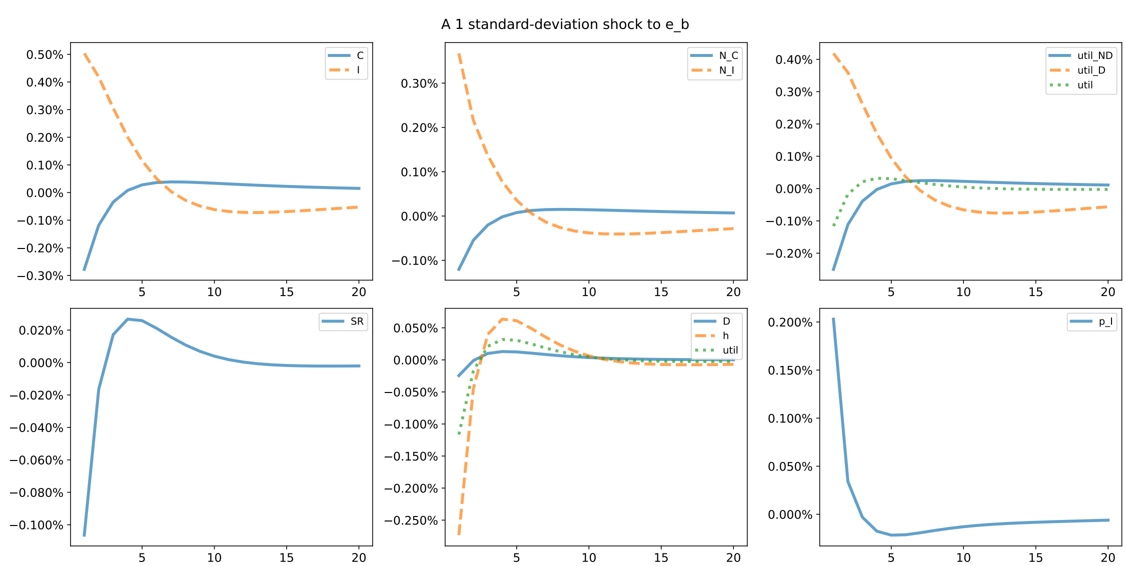

Figure 6: A unit standard deviation negative shock e_b to the discount factor in baseline model with parameters set at the posterior mean. The outcome variables are presented in growth rates.

What about technology shocks? It may appear that technology shocks can generate all the comovement properties as search demand shocks. To that end, Figure 7 plots the impulse response to a unit standard deviation neutral stationary technology shock. The Solow residual rises, but by a smaller amount than from the demand shock. The shock generates positive comovement in consumption and investment, as well as in the labor input of each sector. Thus, a positive technology shock is consistent with sectoral comovement as described by Christiano and Fitzgerald (1998) and Katayama and Kim (2018). Limited factor mobility contributes to this feature. Moreover, the relative price of investment falls. However, utilization in nondurables, part of the consumption sector, actually falls before rising. The technology boost increases the expected return on investment, thereby incentivizing an immediate rise in utilization in the durable sector. After a few periods, the effects of the technology shock subside, and utilization in nondurables respond positively. Hence, search demand shocks are unique in producing positive comovement in the growth rates of all series.

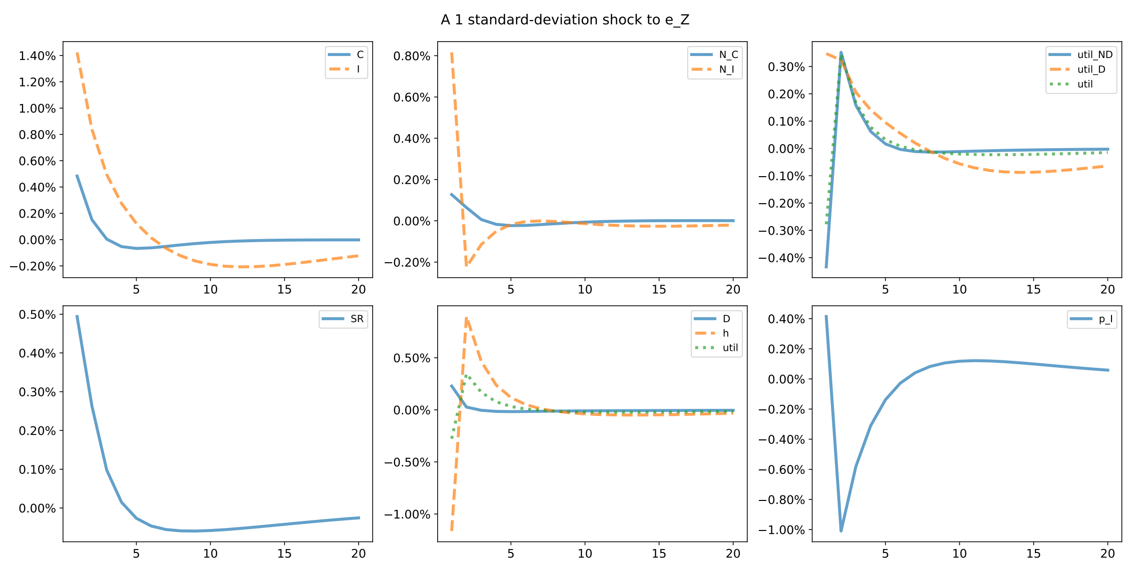

Figure 7: A unit standard deviation negative shock e_z to technology in the baseline model with parameters set at the posterior mean. The outcome variables are presented in growth rates.

7. Conclusion

Using Bayesian methods, we estimate a three-sector model in which goods market frictions allow for shopping effort to influence the Solow residual. Actual output is generally below capacity due to matching frictions. Search-based demand shocks therefore influence capacity utilization which in turn affects the Solow residual. To estimate the model, we adopt a novel approach that utilizes sectoral data on capacity utilization in both the nondurables and durables sectors, alongside data on labor hours and output in the consumption and investment sectors. This unique combination of data allows us to incorporate information on sectoral productivity while also subjecting the model to a rigorous test. We ask the model to not only fit dynamics of capacity utilization overall but also to capture the comovement and persistence of its subcomponents.

We estimate high and precise values of the matching function elasticity ϕ with respect to shopping effort and establish that shocks to shopping effort account for a large part of the forecast error variance of output, the Solow residual, the relative price of investment, hours, and utilization. In terms of empirical fit, the model explains comovement in labor input, output, and utilization well; and also mostly matches the volatility of the utilization series. We also find that key parameters are well-identified by estimating the model on artificial data, drawn at the posterior mean, and showing that parameter estimates are clustered around the true values.

We examine in detail the contribution of different model ingredients. The core findings persist without fixed costs or limited factor mobility—eliminating fixed costs actually enhances the marginal likelihood. However, sector-specific wage markups and search-based demand shocks prove essential for accurate sectoral data fitting. Models restricted to common wage markup shocks significantly overestimate volatility and fail to replicate labor-utilization comovement patterns. Without search-based demand shocks, shopping-effort variables become overdetermined, constrained simultaneously by both output variables with relative price and by utilization variables. Estimating the model without search-based demand shocks and utilization variables fits standard macro series well but generates a counterfactual negative correlation of the utilization variables and understates their volatility.

More broadly, we have exploited sectoral data to argue in favor of a demand-based explanation of the business cycle. Like Bai, Rios-Rull, and Storesletten (2024), we have deliberately abstracted from nominal rigidity to isolate the search-based demand channel, thus not relying on monetary policy transmission or using monetary variables in estimation. This choice actually tilts the playing field in favor of technology shocks. In particular, we do not impose the finding from the literature that technology shocks reduce labor input in the short run (Gali (1999), Basu, Fernald, and Kimball (2006), Francis and Ramey (2005)). Nonetheless, a monetary context would be valuable by incorporating data on inflation and interest rates and using them to to discipline demand shocks. It would also link capacity utilization, an observed notion of economic slack, to the output gap, the latent notion of slack in New Keynesian models.

Our demand shocks include a standard shock to the discount factor (θ_b) and two novel shocks related to goods market frictions (θ_d and θ_i), with the latter proving significantly more influential for business cycle fluctuations. Of course, we do not mean to literally imply that the relevant demand shocks necessarily involve fluctuations in shopping disutility. A key requirement is the ability to explain the main comovement features of the data, including those of capacity utilization. A valuable application for future work would be to incorporate shocks to confidence, as in Angeletos, Collard, and Dellas (2018), in a setting with goods market frictions and endogenous shopping effort. Linking autonomous movements in confidence with shopping effort is in the spirit of Keynes (1936) but is conceptually distinct from the New Keynesian paradigm.

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Appendix A. Background on endogeneity of TFP

The early real business cycle literature treated the Solow residual as a pure measure of technology, but subsequent analysis found that it contained important components unrelated to technology. To address this issue, Basu, Fernald, and Kimball (2006) purify the Solow residual by removing aggregation effects, variation in capital and labor utilization, non-constant returns to scale, and imperfect competition. They find that the purified technology process is about half as volatile as TFP, appears to be permanent, and is generally uncorrelated with output. Building on these findings, Fernald (2014) constructs a quarterly measure of TFP adjusted for utilization. Figure A.8 plots detrended utilization-adjusted TFP alongside standard TFP. The Fernald series not only leads the Solow residual but also exhibits less volatility. Moreover, these series occasionally diverge significantly, most notably during the pandemic shock in 2020Q1, the Great Recession, and the recession of the early 1980's. For what follows, we define Fernald utilization as the difference between cyclical TFP and its utilization-adjusted counterpart.

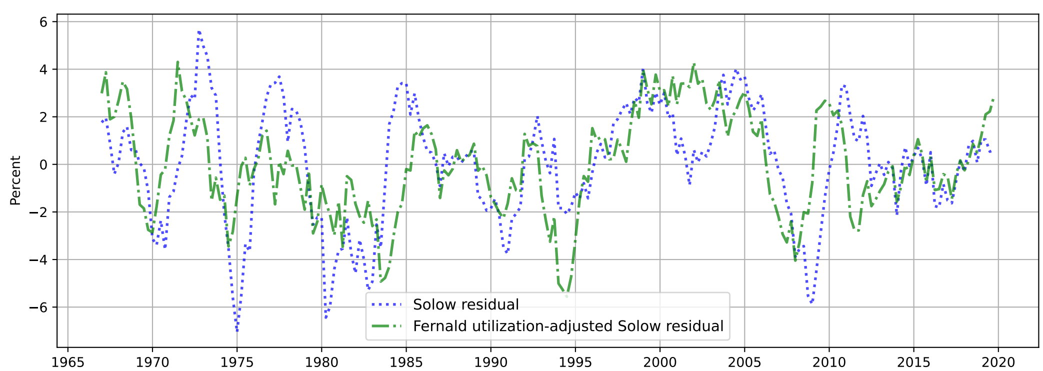

Figure A.8: Time series of the Solow residual and its utilization-adjusted counterpart, following the methodology in Fernald (2014). Each underlying series is detrended via the Hamilton regression filter with the four most recent observations 8 quarters in the past ($p = 4, h = 8$)

Appendix B. Data appendix

Table B.9 provides the details on constructing the model variables, which are used for summary statistics and Bayesian estimation.

Symbol	Description	Construction
C	Nominal consumption	PCND+PCESV
I	Nominal gross private domestic investment	PCDG+PNFI+PRFI
$Deflator$	GDP Deflator	GDPDEF
Pop	Civilian non-institutional population	CNP160V
P_c	Price index: consumption	PCEPI
P_i	Price index: investment	INVDEV
c	Real per capita consumption	$\frac{C}{Pop*P_c}$
i	Real per capita investment	$\frac{I}{Pop*P_i}$
y	Real per capita output	$c+i$
n_c	Labor in consumption sector	Labor in nondurables and services
n_i	Labor in investment sector	Labor in construction and durables
n	Aggregate labor	n_c+n_i
p_i	Relative price of investment	P_i/P_c
$util_{ND}$	Total capacity utilization: nondurables	TCU
$util_D$	Total capacity utilization: durables	TCU
SR	Solow residual	Fernald (2014), FRB of San Francisco
SR_{util}	Utilization-adjusted Solow residual	Fernald (2014), FRB of San Francisco

Table B.9: Data sources used in motivating evidence and estimation.

The construction of sectoral data follows Katayama and Kim (2018). We obtain consumption and investment as follows:

$$C_t = \left(\frac{Nondurable(PCND) + Services(PCESV)}{P_c \times CivilianNonstitutionalPopulation(CNP160V)} \right)$$

$$I_t = \left(\frac{Durable(PCDG) + NoresidentialInvestment(PNFI) + ResidentialInvestment(PRFI)}{P_i \times CivilianNoninstitutionalPopulation(CNP160V)} \right)$$

We use an HP-filtered trend for population ($\lambda = 10,000$) to eliminate jumps around census dates.

For labor data, we make use of the BLS Current Employment Statistics (https://www.bls.gov/ces/data). BLS Table B6 contains the number of production and non-supervisory employees by industry, and BLS Table B7 contains average weekly hours of each sector. We compute total hours for nondurables, services, construction, and durables by multiplying the relevant components of each table. Then we impute labor in consumption as sum of labor in nondurables and services. Similarly, we construct labor in investment as sum of labor in construction and durables. Figure B.9 plots labor hours in each sector. The close comovement and greater volatility of hours in investment is evident.

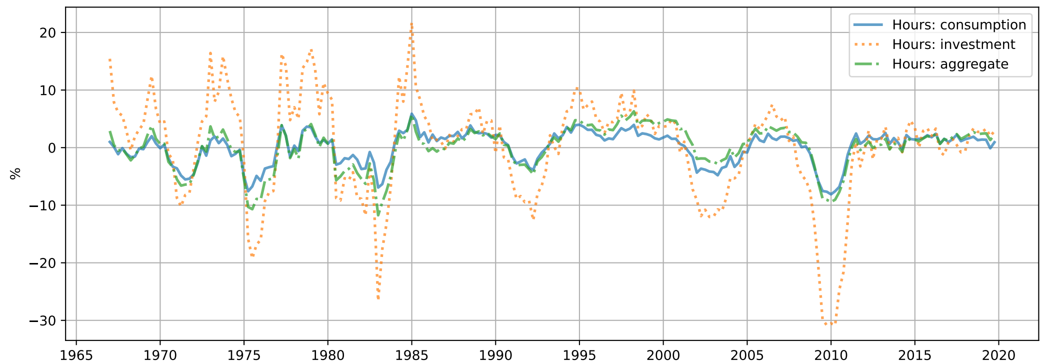

Figure B.9: Sectoral and aggregate hours. Each underlying series is detrended via the Hamilton regression filter with the four most recent observations 8 quarters in the past ($p = 4, h = 8$).

We also make use of disaggregated data on total capacity utilization from the Federal Reserve Board. Estimates are available for 89 detailed industries (71 manufacturing, 16 mining, 2 utilities) and also for several industry groups. Our focus is on durables and nondurables. The data can be downloaded at https://www.federalreserve.gov/datadownload/Choose.aspx?rel=G17.

Appendix C. Details of household and firm problem

Competitive search creates additional interconnections between the household and firm problems. A complete characterization requires solving both jointly. We start with the household problem. Let $\gamma_{mc}, \gamma_{sc}, \gamma_i, \lambda, \mu_{mc}, \mu_{sc}, \mu_i$ be the respective Lagrangian multipliers on the constraints. The first order conditions are

$$[y_{mc}]: \quad u_{mc} = \gamma_{mc} + \lambda p_{mc}$$

$$[y_{sc}]: \quad u_{sc} = \gamma_{sc} + \lambda p_{sc}$$

$$[i_j]: \quad -\gamma_i - \lambda p_j + \mu_j \left(1 - S_j'(x_j)x_j - S_j(x_j)\right) + \beta\theta_b \mathbb{E}\mu_j' S_j'(x_j')(x_j')^2 = 0 \tag{C.1}$$

$$[d_j]: \quad u_d = -A_j D_j^{\phi-1} F_j \gamma_j, \quad j \in \{mc, sc\} \tag{C.2}$$

$$[d_i]: \quad u_d\theta_i = -A_i D_i^{\phi-1} F_i \gamma_i \tag{C.3}$$

$$[n_c]: \quad u_n \frac{\partial n^a}{\partial n_c} = -\lambda W_c^* \tag{C.4}$$

$$[n_i]: \quad u_n \frac{\partial n^a}{\partial n_i} = -\lambda W_i^* \tag{C.5}$$

$$[h_j] \quad \delta_h(h_j)\mu_j = \lambda R_j \quad j \in \{mc, sc, i\} \tag{C.6}$$

$$[k_j']: \quad \mu_j = \beta\theta_b \mathbb{E}\left\{\lambda' R_j' h_j' + (1-\delta_j(h_j'))\mu_j'\right\} \quad j \in \{mc, sc, i\} \tag{C.7}$$

The multipliers $\gamma_{mc}, \gamma_{sc}, \gamma_i$ reflect the value of an additional unit of traded output. In the consumption submarkets, these represent a wedge between the marginal utility of consumption and the marginal utility of wealth. For investment, the multiplier γ_i represents an analogous wedge between the marginal utility of wealth and value of the investment good. Equations (C.2) and (C.3) equate the marginal shopping disutility to the additional units obtained by search multiplied by the value of the unit. Equations (C.4) and (C.5) equate the marginal disutility of work in each sector to the (variable) wage multiplied by the marginal utility of wealth. Equation (C.6) equates the marginal cost of depreciated capital to the value of additional output generated in terms of consumption. Finally, (C.7) equates the marginal value of capital to the expected discounted rate of return, composed of the rental income and value of undepreciated capital.

We next characterize the envelope conditions:

$$\frac{\partial V^j}{\partial p_j} = -\lambda j = -\lambda d_j A_j D_j^{\phi-1} F_j \quad j \in \{mc, sc, i\} \tag{C.8}$$

$$\frac{\partial V^j}{\partial D_j} = (\phi-1) d_j A_j D_j^{\phi-2} F_j \gamma_j \quad j \in \{mc, sc, i\} \tag{C.9}$$

$$\frac{\partial V^j}{\partial F_j} = d_j A_j D_j^{\phi-1} \gamma_j \quad j \in \{mc, sc, i\}$$

The ratio of (C.8) and (C.9) characterizes the indifference curve between price and tightness in a submarket:

$$\frac{\frac{\partial V^j}{\partial p^j}}{\frac{\partial V^j}{\partial D^j}} = -\frac{\lambda D_j}{(\phi-1)\gamma_j} \tag{C.10}$$

We next turn to the firm's problem. The firm chooses labor type s in sector j so as to generate an effective labor bundle n_j at the lowest possible cost. The problem is

$$\min_{n_j(s)} \int_0^1 W_j(s) n_j(s) ds \quad \text{s.t.} \tag{C.11}$$

$$\left(\int_0^1 n_j(s)^{1/\mu_j} dj \right)^{\mu_j} \geq \overline{n} \tag{C.12}$$

Take the first order condition of (C.11) and recognize W_j as the Lagrangian multiplier on constraint (C.12). Rearrange as

$$n_j(s) = \left(\frac{W_j(s)}{W_j} \right)^{-\frac{\mu_j}{\mu_j - 1}} n_j$$

The corresponding wage index for composite labor input in sector j is

$$W_j = \left[\int_0^1 W_j(s)^{1/(\mu_j - 1)} ds \right]^{\mu_j - 1}$$

We can now examine the simplified firm problem. Let ι_j and ∇_j be the multipliers on participation constraint and production technology. The first order conditions are

$$[F_j] \quad \nabla_j = p_j A_j D_j^{\phi} + \iota_j \frac{\partial V^j}{\partial F^j}$$

$$[n_j] \quad W_j = \nabla_j z_j f_n \tag{C.13}$$

$$[k] \quad h_j R_j = \nabla_j z_j f_k \tag{C.14}$$

$$[p_j] \quad A_j D_j^{\phi} F_j + \iota_j \frac{\partial V^j}{\partial p_j} = 0 \tag{C.15}$$

$$[D_j] \quad \phi A_j D_j^{\phi - 1} p_j F_j + \iota_j \frac{\partial V^j}{\partial D^j} = 0 \tag{C.16}$$

Take the ratio of first order conditions (C.15) and (C.16) to alternately characterize the indifference curve between price and tightness:

$$\frac{\frac{\partial V^j}{\partial p^j}}{\frac{\partial V^j}{\partial D^j}} = \frac{D_j}{\phi p_j}$$

Plug in (C.10) to find

$$\frac{D_j}{\phi p_j} = -\frac{\lambda D_j}{(\phi - 1)\gamma_j}$$

which we rearrange as

$$\gamma_j = \frac{\phi}{1-\phi}\lambda p_j$$

Since $\gamma_j = u_j - \lambda p_j$ for $j = \{mc, sc\}$, we have

$$\lambda = (1-\phi)\frac{u_j}{p_j} \tag{C.17}$$

which allows us to characterize γ_i:

$$\gamma_i = \phi\frac{u_j}{p_j}p_i \quad j \in \{mc, sc\}$$

Note that (C.17) also implies that the marginal utility relative to the price is the same in each consumption subsector. The values of γ_{mc}, γ_{sc} and λ allows us to rewrite the shopping optimality conditions and labor leisure tradeoff:

$$\begin{aligned}
-u_d &= \phi u_j A_j D_j^{\phi-1}[z_j f(h_j k_j, n_j) - \nu_j] \quad j \in \{mc, sc\} \\
-u_d\theta_i &= \phi\frac{u_{mc}p_i}{p_{mc}}A_i D_i^{\phi-1}[z_i f(h_i k_i, n_i) - \nu_i] \\
u_n\frac{\partial n^a}{\partial n_j} &= -\frac{u_{mc}(1-\phi)}{p_{mc}}W_j^* \quad j \in \{c, i\}
\end{aligned}$$

We next revisit the investment first order condition (C.1) and characterize Tobin's Q. For sector $j \in \{mc, sc, i\}$ we have

$$\begin{aligned}
\lambda p_i + \gamma_i &= \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b\mathbb{E}\mu_j'(S'(x_j')(x_j')^2) \\
\lambda p_i + \frac{\phi}{1-\phi}\lambda p_i &= \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b\mathbb{E}\mu_j'(S'(x_j')(x_j')^2) \\
\frac{\lambda p_i}{1-\phi} &= \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b\mathbb{E}\mu_j'(S'(x_j')(x_j')^2)
\end{aligned}$$

Let $Q_j = \mu_j/\lambda$: relative price of capital in sector j in terms of consumption. Using Q_j rewrite the choice of optimal investment as

$$\frac{p_i}{1-\phi} = Q_j[1 - S_j'(x_j)x_j - S_j(x_j)] + \beta\theta_b\mathbb{E}\frac{\lambda'}{\lambda}Q_j'S_j'(x_j')(x_j')^2$$

We also use Tobin's Q to rewrite the optimal utilization in $j \in \{mc, sc, i\}$ and the Euler equation:

$$\begin{aligned} \delta_h(h_j)Q_j &= R_j \\ Q_j &= \beta\theta_b \mathbb{E}\frac{\lambda'}{\lambda}\left[(1-\delta(h_j'))Q_j' + R_j'h_j'\right] \end{aligned}$$

It remains to solve for the Lagrangian multipliers ι_j and ∇_j on the firm problem. This is straightforward given λ and γ_j. First,

$$\iota_j = \frac{A_j q_j^\phi F_j}{\frac{\partial V^j}{\partial p_j}} = \frac{1}{\lambda}$$

Second,

$$\begin{aligned} \nabla_j &= p_j A_j D_j^\phi + \iota_j \frac{\partial V^j}{\partial F^j} \\ &= p_j A_j D_j^\phi + \frac{A_j D_j^\phi \gamma_j}{\lambda} \\ &= p_j A_j D_j^\phi + A_j D_j^\phi \frac{\phi}{1-\phi} p_j \\ &= A_j D_j^\phi \left(p_j + \frac{\phi}{1-\phi} p_j\right) \\ &= \frac{p_j A_j D_j^\phi}{1-\phi} \end{aligned}$$

The value of additional production capacity ∇_j exceeds the additional sales $p_j A_j D_j^\phi$. This is because the additional sales also relax the participation constraint of households. Finally, the value of these multipliers enables us to characterize the factor demands for the firms. Substitute for ∇_j in (C.13) to find

$$\begin{aligned} (1-\phi)\frac{W_j}{p_j} &= A_j (D_j)^\phi z_j \frac{\partial f(h_j k_j, n_j)}{\partial n} \\ &= \frac{\alpha_n}{n_j} A_j D_j^\phi z_j f(h_j k_j, n_j) \\ &= \frac{\alpha_n}{n_j} A_j D_j^\phi \left(\frac{y_j}{A_j D_j^\phi} + \nu_j\right) \\ &= \frac{\alpha_n}{n_c}(y_j + A_j D_j^\phi \nu_j) \end{aligned}$$

$$= \frac{\alpha}{n_c} y_j (1 + \nu^R)$$

where we use $\nu_j^R = \nu_j \Psi_T / y_j$. We can simplify the capital demand (or rental rate) (C.14) using ratios as

$$\frac{W_j}{R_j} = \frac{\alpha_n}{\alpha_k} \frac{h_j k_j}{n_j}$$

Aggregating across sectors, the steady-state labor labor of income is $\alpha_n (1 + \nu^R)/(1 - \phi)$ and the capital share of income is $\alpha_k (1 + \nu^R)/(1 - \phi)$.

Appendix D. Calibration

In general, we determine some (fixed) parameters from long-run targets, estimate the parameter set Θ described in the main text, and back out the remaining (dependent parameters) given draws from Θ and long-run targets. The dependent parameters are thus random variables. Here we use the term calibration more broadly to characterize the determination of dependent parameters as a function of both estimated parameters and long-run targets.

Several key targets used for calibration are investment-to-output $p_i I/Y$, capital-to-output $p_i k/Y$, the labor share of income, the unconditional growth rate $\overline{g}$, and share of services S_c in consumption. In terms of model variables at quarterly frequency, we have

$$\kappa \equiv p_i I/Y = 20\%,\ p_i k/Y = 2.75(4) = 11,\ \overline{g} = 0.45\%,\ \tau \equiv \frac{nW}{Y} = 67\%,\ S_{sc} \equiv \frac{p_{sc} y_{sc}}{C} = 65\%$$

The first two targets are identical to Bai, Rios-Rull, and Storesletten (2024), and the third corresponds to 1.8% per capital annual growth, which is very close to the average over the data sample. Capital accumulation (ignoring adjustment costs) in transformed variables [18] is given by

$$g \widehat{k}' = (1 - \delta) \widehat{k} + g \widehat{I}$$

[18]Investment is divided by the stochastic trend $\hat{I}_t = I_t / X_t$ while the capital stock is divided by the lagged stochastic trend $\hat{K}_t = K_t / X_{t-1}$ to maintain its status as a predetermined variable.

Balanced growth, in terms of original variables, implies a steady state in terms of $\widehat{k}$, such that

$$\delta = 1 - \overline{g} + \frac{I}{k} \approx 1.37\%$$

Next, we characterize α_n, α_k and σ_b. Labor demand (18) for each sector implies

$$W_j n_j = \frac{\alpha_n}{1-\phi} p_j Y^j (1 + \nu_j^R)$$

where $\nu_j^R = \nu_j X / F_j$. The steady state labor share is thus

$$\frac{\sum W_j n_j}{Y} = \frac{\alpha_n}{1-\phi} \frac{C + p_i I}{Y} (1 + \nu_{ss}^R) = \frac{\alpha_n}{1-\phi} (1 + \nu_{ss}^R)$$

so that $\alpha_n = (1-\phi)\text{labor share}/(1+\nu_{ss}^R)$.

In steady state, the rate of return on capital in each sector is equal, so we let R denote the common value: $R = R_j$ for all j. It is helpful to use the interest rate r on an illiquid bond as the value which satisfies $\beta \overline{g}^{-\sigma} = 1/(1+r)$.

The Euler equations in the steady state imply

$$\begin{aligned} Q &= \beta \overline{g}^{-\sigma} [(1-\delta) Q + R] \Rightarrow \\ (1+r) Q &= (1-\delta) Q + R \\ (r+\delta) Q &= R \end{aligned}$$

Given that capital utilization $h_j = 1$ for all j in the steady state, the parameter σ_b satisfies

$$\sigma_b = \frac{R}{Q} = r + \delta$$

Combining with Tobin's Q, $p_i/(1-\phi) = Q$, we have

$$(1-\phi) \frac{R}{p_i} = r + \delta$$

Now, turn to the firm demand for capital (19):

$$(1-\phi)\frac{R_j}{p_j} = \alpha_k \frac{Y_j}{k_j}(1+\nu^R)$$

An immediate corollary is that $Y_j/k_j = Y/k$ for all k and hence

$$r+\delta = \alpha_k \frac{Y}{k}(1+\nu^R)$$

so that

$$\alpha_k = \frac{r+\delta}{1+\nu^R}\frac{k}{Y}$$

We pin down the weight of services ω_{sc} as the empirical measure $S_c = p_{sc}Y_{sc}/C$ and set $S_c = 0.65$. The ratio of demand in consumption subsectors implies

$$\frac{Y_{mc}}{Y_{sc}} = \left(\frac{p_{mc}}{p_{sc}}\right)^{-\xi} \frac{\omega_{mc}}{\omega_{sc}}$$

Multiply each side by p_{mc}/p_{sc}, so that

$$\frac{p_{mc}Y_{mc}}{p_{sc}Y_{sc}} = \left(\frac{p_{mc}}{p_{sc}}\right)^{1-\xi} \frac{\omega_{mc}}{\omega_{sc}}$$

and plug in S_c:

$$\left(\frac{1-S_c}{S_c}\right) = \left(\frac{p_{mc}}{p_{sc}}\right)^{1-\xi} \frac{1-S_c}{S_c}$$

so that $p_{mc} = p_{sc}$. Since we normalize $p_{sc} = 1$ and have also normalized the consumption price index to unity, we have $p_{mc} = p_{sc} = p_c = 1$.

Given the target for capacity utilization $\Psi_{T,j}$, we wish to find the corresponding level coefficient $A_j = \Psi_{T,j}/D_j^{\phi}$. This entails solving for each D_j. We first solve for D. Let us sum each side of the shopping optimality condition across sectors:

$$\sum_j D^{1/\eta} D_j = \sum_j \phi p_j Y_j$$
$$D^{\frac{\eta+1}{\eta}} = \phi Y$$

Given that we choose technology coefficients such that $Y = 1$, we obtain $D = \phi^{\frac{\eta}{\eta+1}}$.

Now, take the ratio of the shopping conditions rearrange for relative shopping effort:

$$\frac{D_{mc}}{D_{sc}} = \frac{p_{mc}}{p_{sc}} \frac{Y_{mc}}{Y_{sc}} = \frac{1 - S_c}{S_c} \tag{D.1}$$

Similarly,

$$\frac{D_j}{D_i} = S_j \frac{1 - I/Y}{I/Y} \tag{D.2}$$

Now, we put (D.1) and (D.2) together to characterize shopping effort in each sector:

$$\begin{aligned} D_{mc} &= (1 - S_c)(1 - I/Y)D \\ D_{sc} &= S_c(1 - I/Y)D \\ D_i &= (I/Y)D \end{aligned}$$

Appendix E. Cyclical deviations of Solow residual and total capacity utilization

In the main text we analyze the relationship between the Solow residual and capacity utilization in growth rates. Here we compare them in terms of cyclical deviations. Using (22), the cyclical component of the Solow residual is

$$\hat{SR}_j \equiv \frac{SR_j}{X^\tau} = \frac{A_j D_j^\phi z_j h_j^{\alpha_k} g^{1-\alpha_k-\tau} \hat{k}_j^{\alpha_k-1+\tau} n_j^{\alpha_n-\tau}}{1 + \nu_j^R} = g^{1-\tau} \frac{\hat{Y}_j}{\hat{k}_j^{1-\tau} n_j^\tau}$$

The log linear representation is

$$\widetilde{\hat{SR}}_j = \phi \widetilde{D}_j + \widetilde{z}_j + \alpha_k \widetilde{h}_j + (1 - \alpha_k - \tau)\widetilde{g} + (\alpha_k - 1 + \tau)\widetilde{\hat{k}}_j + (\alpha_n - \tau)\tilde{n}_j - \frac{\nu_{ss}^R}{1 + \nu_{ss}^R} \widetilde{\nu}_j^R$$

and note that $\widetilde{g}_t = \log g_t - \log \overline{g}$ which is first-order equivalent to X^{obs}. Log linearizing (25) yields

$$\widetilde{util}_j = \phi \widetilde{D}_j + (1 + \nu_{ss}^R)\alpha_k \widetilde{h}_j$$

Thus, in the absence of fixed costs, we have

$$\widetilde{\hat{SR}}_j|_{\nu_j=0} = \widetilde{util}_j + \tilde{z}_j + (1 - \alpha_k - \tau)(\log g_t - \log \overline{g}) + (\alpha_k - 1 + \tau)\tilde{\hat{k}}_j + (\alpha_n - \tau)\tilde{n}_j$$

Given the detrending, the coefficient on nonstationary technology is $1 - \alpha_k - \tau$ rather than $1 - \alpha_k$. Otherwise, the relationship between cyclical components of the Solow residual and utilization has the same form as the one in growth rates.

The relationship between the cyclical form and growth rate form is

$$\begin{aligned} dSR_t &= \Delta \log SR_t \\ &= \log \hat{SR}_t + \tau \log X_t - (\log \hat{SR}_{t-1} + \tau \log X_{t-1}) \\ &= \Delta \widetilde{\hat{SR}}_{jt} + \tau \log g_t \end{aligned}$$

The growth rate of the Solow residual equals the growth rate of cyclical deviations plus the log deviation of the stochastic trend growth rate relative to the unconditional mean multiplied by the labor share.

Appendix F. Stochastic singularity in the absence of search demand shocks for special case of the model

We find numerically in Table 8 that search-based demand shocks are essential to fit the data. Here we show that, in a special case of the model, the absence of search demand shocks actually gives rise to stochastic singularity. That is, an observable series is a deterministic function of other observable series and predetermined variables. Since full-information methods require one to match the entire observed series for some sequence of shocks, this renders estimation impossible under this approach.

Specifically, consider a unitary consumption sector and abstract from fixed costs and investment adjustment costs. Then equation (20) becomes

$$\frac{p_i}{p_c} = \frac{n_i W_i}{n_j W_j} \frac{C}{I}$$

Absent search demand shocks, (21) simplifies to

$$\frac{D_i}{D_c} = \frac{1}{\theta_i}\frac{n_i W_i}{n_c W_c}$$

and hence

$$\frac{D_i}{D_c} = \frac{p_i}{p_c}\frac{I}{C} \tag{F.1}$$

From (F.1), the shopping effort ratio is entirely pinned down in terms of observables. Utilization in this special case satisfies

$$util_j = A_j D_j^{\phi} h_j^{\alpha_k} \quad j \in \{c, i\} \tag{F.2}$$

Since, for each j, D_j is pinned down by observables, stochastic singularity arises if h_j is also pinned down by observables.

Recall that optimal utilization has the form $\delta_h^j(h_j)Q_j = R_j$ for $Q_j = p_j/(1-\phi)$. Moreover, we can express the rental of capital as $R_j = \alpha_k Y_j/(h_j K_j)$ and hence

$$\delta_h^j(h_j)\frac{p_j}{1-\phi} = \alpha_k \frac{Y_j}{h_j k_j}$$

so that h_j is a function of observables and predetermined capital. Consequently, using (F.2), utilization in each sector is a function of other observables, and there is stochastic singularity.

Appendix G. Equilibrium in basic BRS model

Given initial states $\{k_{c0}, k_{i0}\}$ and $\{g_0, \theta_{d0}, \theta_{n0}, z_{c0}, z_{i0}\}$, an equilibrium is a sequence of prices $\{p_{it}, R_{ct}, R_{it}, W_t\}_{t=0}^{\infty}$ and quantities $\{k_{ct}, k_{it}, k_t, C_t, I_t, D_{ct}, D_{it}, D_t, n_{ct}, n_{it}, n_t, g_t, \theta_{dt}, \theta_{nt}, z_{ct}, z_{It}\}_{t=0}^{\infty}$ which solve the following system given the realization of shocks $\{e_{gt}, e_{vt}\}_{t=0}^{\infty}$:

$$\theta_{nt} n_t^{1/\nu} = (1-\phi)W_t$$

$$\theta_{dt} D_t^{1/\eta} = \phi \frac{C_t}{D_{ct}}$$

$$\theta_{dt} D_t^{1/\eta} = \phi p_{it} \frac{I_t}{D_{it}}$$

$$\Gamma_t = C_t - \theta_{dt}\frac{D_t^{1+1/\eta}}{1+1/\eta} - \theta_{nt}\frac{(n_t)^{1+1/\zeta}}{1+1/\zeta}$$

$$\Gamma_t^{-\sigma} p_{it} = \beta\mathbb{E}\left\{[(1-\phi)R_{c,t+1} + p_{i,t+1}(1-\delta)](\Gamma_{t+1}g_{t+1})^{-\sigma}\right\}$$

$$\mathbb{E}(R_{c,t+1} - R_{i,t+1}) = 0$$

$$C_t = A_c(D_{ct})^{\phi} z_{ct} g_t^{-\alpha_k} k_{ct}^{\alpha_k} n_{ct}^{\alpha_n}$$

$$I_t = A_i(D_{it})^{\phi} z_{it} g_t^{-\alpha_k} k_{it}^{\alpha_k} n_{it}^{\alpha_n}$$

$$I_t g_t = (k_{c,t+1} + k_{i,t+1})g_t - (1-\delta)(k_{ct} + k_{it})$$

$$(1-\phi)\frac{W_t}{p_t} = \alpha_n \frac{C_t}{n_{ct}} \quad j \in \{c, i\}, \quad \text{with} \quad p_{ct} = 1$$

$$\frac{W_t}{R_{jt}} = \frac{\alpha_n}{\alpha_k}\frac{k_{jt}}{n_{jt}} \quad j \in \{c, i\}$$

$$n_t = n_{ct} + n_{it}, k_t = k_{ct} + k_{it}, D_t = D_{ct} + D_{it}$$

$$\log g_t = (1-\rho_g)\overline{g} + \rho_g \log g_{t-1} + e_{gt}$$

$$\log v_t = \rho_v \log v_{t-1} + e_{v,t}, v \in \{\theta_d, \theta_n, z_c, z_I\}$$

$$\log z_{it} = \log z_{ct} + \log z_{It}$$

Appendix H. Equilibrium of baseline model

Given initial states $\{k_{mc0}, k_{sc0}, k_0\}$ and $\{g_0, \theta_{b0}, \theta_{d0}, \theta_{i0}, \theta_{n0}, z_{c0}, z_{I0}, \mu_{c0}, \mu_{i0}\}$, an equilibrium is a sequence of prices $\{p_{it}, R_{jt}, Q_{jt}, W_{ct}, W_{it}\}_{t=0}^{\infty}$ and quantities $\{k_{jt}, i_{jt}, Y_{jt}, C_t, D_{jt}, n_t^a, n_{jt}, n_{ct}, n_t, g_t, \theta_{bt}, \theta_{dt}, \theta_{it}, \theta_{nt}, z_{ct}, z_{It}, \mu_{ct}, \mu_{it}\}_{t=0}^{\infty}$ for $j \in \{mc, sc, i\}$ that solves the following system given the realization of shocks $\{e_{gt}, e_{vt}\}_{t=0}^{\infty}$:

$$\theta_n (n_t^a)^{1/\nu} \left(\frac{n_{ct}}{n_t^a}\right)^{\theta} \omega^{-\theta} = (1-\phi)\frac{W_{ct}}{\mu_{ct} S_t}$$

$$\theta_n (n_t^a)^{1/\nu} \left(\frac{n_{it}}{n_t^a}\right)^{\theta} (1-\omega)^{-\theta} = (1-\phi)\frac{W_{it}}{\mu_{it} S_t}$$

$$n_t^a = \left[\omega^{-\theta} n_{ct}^{1+\theta} + (1-\omega)^{-\theta} n_{it}^{1+\theta}\right]^{\frac{1}{1+\theta}}$$

$$\Gamma_t = C_t - \theta_{dt}\frac{D_t^{1+1/\eta}}{1+1/\eta} - \theta_{nt}\frac{(n_t)^{1+1/\zeta}}{1+1/\zeta}$$

$$\theta_{dt} D_t^{1/\eta} = \phi p_{jt}\frac{Y_{jt}}{D_{jt}} \quad j \in \{mc, sc\}$$

$$\theta_{it}\theta_{dt} D_t^{1/\eta} = \phi p_{it}\frac{I_t}{D_{it}}$$

$$\frac{p_{it}}{1-\phi} = Q_{jt}[1 - S'(x_{jt})x_{jt} - S_j(x_{jt})] + \beta\theta_b \mathbb{E}_t\left(\frac{\Gamma_{t+1}}{\Gamma_t}\right)^{-\sigma} g_{t+1}^{-\sigma} Q_{j,t+1} S'(x_{j,t+1})(x_{j,t+1})^2 \quad j \in \{mc, sc, i\}$$

$$Q_{jt} = \beta\theta_{bt}\mathbb{E}t\left(\frac{\Gamma_{t+1}}{\Gamma_t}\right)^{-\sigma} g_{t+1}^{-\sigma}\left[(1-\delta_j(h_{j,t+1}))Q_{j,t+1} + R_{j,t+1}h_{j,t+1}\right] \qquad j \in \{mc, sc, i\}$$

$$C_t = [\omega_c^{1-\rho_c} Y_{mc,t}^{\rho_c} + (1-\omega_c)^{1-\rho_c} Y_{sc,t}^{\rho_c}]^{1/\rho_c}$$

$$Y_{jt} = p_{jt}^{-1/(1-\rho_c)}\omega_j C_t \quad j \in \{mc, sc, i\}$$

$$C_t = p_{mc,t}Y_{mc,t} + p_{sc,t}Y_{sc,t}$$

$$\delta_h(h_{jt})Q_{jt} = R_{jt}, \quad j \in mc, sc, i$$

$$Y_{jt} = A_j(D_{jt})^{\phi}(z_{jt}g_t^{-\alpha_k}(h_{jt}k_{jt})^{\alpha_k}(N_{jt})^{\alpha_n} - \nu_j) \quad j \in \{mc, sc, i\}$$

$$k_{j,t+1}g_t = (1-\delta_j(h_{jt}))k_{jt} + [1 - S_j(x_{jt})]I_{jt}g_t \quad j \in \{mc, sc, i\}$$

$$(1-\phi)\frac{W_{jt}}{p_{jt}} = \alpha_n\frac{Y_{jt} + A_j D_{jt}^{\phi}\nu_j}{n_{jt}} \quad j \in \{mc, sc, i\}$$

$$\frac{W_{jt}}{R_{jt}} = \frac{\alpha_n}{\alpha_k}\frac{h_{jt}k_{jt}}{n_{jt}} \quad j \in \{mc, sc, i\}$$

$$n_{ct} = n_{mct} + n_{sct}, n_t = n_{ct} + n_{it}, D_t = D_{mct} + D_{sct} + D_{it}$$

$$k_t = k_{mct} + k_{sct} + k_{it}, I_t = I_{mct} + I_{sct} + I_{it}$$

$$\log g_t = (1-\rho_g)\log\overline{g} + \rho_g \log g_{t-1} + e_{g,t}$$

$$\log v_t = \rho_v \log v_{t-1} + e_{v,t}, \quad v \in \{\theta_b, \theta_d, \theta_n, \theta_i, z_c, z_I, \mu_c, \mu_i\}$$

Appendix I. The forecast error variance decomposition for specific demand and technology shocks

Here we decompose the variance decomposition of demand and technology shocks. The main takeaway from Table I.10 is that neutral search demand shocks dominate the forecast error variance of all variables except for the relative price of investment. In particular, it accounts for over 96% of the demand component of utilization.

Table I.10: Forecast error variance decomposition

	e_d	e_{di}
Y	97.23	2.77
SR	94.26	5.74
I	88.83	11.17
p_i	46.65	53.35
n_c	99.67	0.33
n_i	96.38	3.62
$util$	96.92	3.08
D	99.97	0.03
h	98.77	1.23

Table I.10: Contribution of components to forecast error variance decomposition of search shocks.

In a similar vein, I.11, dissects the various constituent elements of technology shocks. Stationary neutral technology shocks e_z are by far the most important overall. However, permanent technology shocks are relatively important for output and especially the Solow residual. Investment-specific technology shocks are, unsurprisingly, important for investment, its relative price, and labor in the investment sector. From both tables it is clear that each is important at explaining at least some aspect of business cycle fluctuations.

Table I.11: Forecast error variance decomposition

	e_g	e_Z	e_{zI}
Y	31.68	63.30	5.02
SR	48.24	43.87	7.90
I	3.25	74.14	22.62
p_i	0.14	43.91	55.95
n_c	22.23	75.51	2.26
n_i	6.20	61.70	32.10
$util$	0.64	83.26	16.10
D	10.20	76.28	13.52
h	1.34	89.29	9.36

Table I.11: Contribution of components to forecast error variance decomposition of technology shocks.

Appendix J. Estimation on artificial data and identification of parameters

To assess the identifiability of key parameters, we conduct an analysis employing artificial data inspired by Schmitt-Grohé and Uribe (2012). This involves setting the parameters at their mean values and following the calibration strategy outlined in Section Appendix D. We generate an artificial dataset comprising 223 observations for each observable variable. Subsequently, we estimate the model using this artificial data, employing the same estimation techniques and prior distributions as in the baseline model.

Table J.12 plots the true value used in generating the artificial data alongside the 5th, 50th, and 95th percentiles of the posterior distribution for each parameter value. We find that the highest posterior density intervals typically contain the true parameter value, often toward the center. In particular, the posterior median for ϕ, 0.782, is very close to 0.752. The parameters associated with search demand shocks $\rho_d, \rho_{di}, e_d, e_{di}$ are also well identified. There is also excellent identification of ha, σ, θ, ξ, and ν_R. The persistence of permanent technology shocks is tougher to identify, as the true value 0.508 lies above the 95th percentile.

Table J.12: Estimation on artificial data

		Posterior distribution		
Parameter	True value	Median	5%	95%
σ	1.711	1.737	1.517	2.001
ha	0.6338	0.6506	0.6083	0.6843
ζ	0.9251	1.001	0.8656	1.133
ϕ	0.7524	0.7815	0.7382	0.825
η	0.3437	0.295	0.2201	0.3943
ξ	0.8655	0.8045	0.6743	0.934
ν_R	0.2443	0.2373	0.1628	0.3118
σ_{ac}	1.897	1.795	1.357	2.262
σ_{ai}	0.4440	0.3317	0.2355	0.4404
Ψ_K	8.364	7.092	6.195	9.292
θ	1.059	1.083	0.9549	1.216
ρ_g	0.5077	0.3264	0.2561	0.4397
ρ_z	0.7427	0.6284	0.5121	0.7305
ρ_{zI}	0.8646	0.8913	0.8440	0.9358
ρ_n	0.9874	0.9517	0.8518	0.9999
ρ_d	0.9244	0.8873	0.8363	0.9334
ρ_{di}	0.9836	0.9259	0.8790	0.9726
ρ_b	0.9280	0.9181	0.8798	0.9555
$\rho_{\mu c}$	0.7912	0.8396	0.4721	0.9852
$\rho_{\mu i}$	0.9804	0.9658	0.9420	0.9889
e_g	0.005891	0.005634	0.005050	0.006281
e_z	0.007803	0.006860	0.006110	0.007650
e_{zI}	0.01491	0.01459	0.01317	0.01618
e_n	0.008313	0.008074	0.006501	0.009594
e_d	0.09850	0.11396	0.08138	0.1385
e_{dI}	0.01728	0.01661	0.01523	0.01807
e_b	0.009097	0.008272	0.003727	0.01412
$e_{\mu c}$	0.0006684	0.001754	0.0001000	0.004045
$e_{\mu i}$	0.02244	0.02128	0.01920	0.02326

Table J.12: We generate artificial data from the model with parameter values equal to the posterior mean of the Bayesian estimation on the actual data, in tandem with the calibration strategy. We then use this artificial data as observables in estimation. The posterior median, 5th percentile, and 95th percentile from the posterior distribution are compared alongside the true values.

Appendix K. Supplementary appendix: Shopping costs in the form of expenditure

Michaillat and Saez (2015) also use matching frictions in the goods market and emphasizes the impact of aggregate-demand shocks on output and employment. At first glance, it is difficult to compare the two settings because Michaillat and Saez (2015) specify the matching frictions differently, formalize matching costs in terms of expenditure rather than disutility, and also incorporate money demand via money in the utility. Accordingly, we represent matching costs in terms of expenditures in a static form of BRS and show that the same key logic holds. However, the labor share of income turns out to be different since expenditure shows up in the national income accounts, but effort does not.

As in the static model in the main text, each firm has a location production function $F = zn^{\alpha_n}$ using just labor. Each unit of search requires an expenditure ρ. In terms of national income accounting, these expenditures are part of consumption, but they yield no utility to households. The remaining part of consumption, c^e, does directly yield utility.

Household preferences take the form $u(c^e, n) = U(\Gamma)$ where U is increasing, strictly concave, and differentiable

$$\Gamma = c^e - \theta_n \frac{n^{1+1/\zeta}}{1+1/\zeta}$$

Thus, there are zero wealth effects on labor supply (GHH).

The link between effective consumption and overall consumption satisfies

$$\begin{aligned} c^e &= C - d\rho \\ &= d(\Psi_d F - \rho) \end{aligned}$$

The necessary units of shopping to consume one service are $1/(\Psi_d F - \rho)$. The associated expenditures are thus

$$T(D) = \frac{\rho}{\Psi_d F - \rho} \tag{K.1}$$

The expression for T in (K.1) differs from the analogue in Michaillat and Saez (2015) only by the fact that the Ψ_d is multiplied by capacity F, which is a consequence of one unit traded per match in their setup.

A household who chooses a particular submarket (p, D) has expenditure $pc^e(1+T(D)) = pC$ and associated income $\pi + nW$, where π denotes firms' profits.

The problem of the household in submarket (p, D) is

$$\max u(c^e, n) \quad s.t.$$
$$pc^e(1 + T(D)) = \pi + nW$$

The first order conditions with respect to c and n yield the following labor-leisure or labor supply condition:

$$\theta_n n^{1/\zeta} = \frac{W/p}{1 + T(D)}$$

The search wedge $1/(1 + T(D))$ reduces the return to working, analogous to a consumption tax or labor income tax.

We next solve the problem of the firm. To keep customers from deviating to another submarket, it must post a combination of price and tightness (p, D) such that $p(1+T(D)) \leq H$ for some H. The problem is

$$\max_{n,p,D} p\Psi_T(D)zn^{\alpha_n} - nW \quad s.t.$$
$$p(1 + T(D)) \leq H$$

The first order condition for n is

$$\alpha_n \frac{\Psi_T F}{n} = W$$

Aggregate consumption satisfies $C = \Psi_T F$, so that $nW/C = \alpha_n$. Hence, the labor share of income is α_n. By contrast, if the matching costs were in terms of disutility, then the corresponding labor share of income would be $\alpha_n/(1-\phi)$.

The problem over the price-tightness pair (p, D) can be simplified by substituting for the constraint in the objective as

$$\frac{\Psi_T(D)}{1+T(D)} = \frac{\Psi_T}{\Psi_D}(\Psi_d F - \rho) = \frac{D}{F}(AD^{\phi-1}F - \rho)$$

Differentiating with respect to D yields

$$\rho = \phi \Psi_D F$$

or, in closed form,

$$D = \left(\frac{\phi A z n^{\alpha_n}}{\rho}\right)^{1/(1-\phi)} \tag{K.2}$$

Notice that (K.2) depends not only on both the parameters of matching technology ϕ, A and cost ρ but also on z and n.

Thus, we normalize $p = 1$ and define equilibrium as a tuple (D, C, c^e, n, W) satisfying

$$\begin{aligned}
\rho &= \phi \Psi_D \\
C &= AD^{\phi} z n^{\alpha_n} \\
c^e &= \frac{C}{1+T(D)} \\
W &= \frac{\alpha_n C}{n} \\
\theta_n n^{1/\zeta} &= \frac{W}{1+T(D)}
\end{aligned}$$

Compared to the baseline setup, the wedge on labor supply is now $1/(1+T(D))$ instead of $1-\phi$ and the labor share of income is α_n. Moreover, the cost of shopping is linear, which is analogous to letting $\eta \to \infty$ in the BRS specification.

A key difference in the labor share of income is that purchased shopping services are still part of GDP. Thus, the Solow residual is $SR = C/n^{\alpha_n} = AD^{\phi}z$. Both matching frictions and technology enter into GDP, but, unlike BRS, there is no input share mismeasurement.

Michaillat and Saez (2015) argue that the effect of aggregate demand shocks on output and employment depends on sticky prices. The reason is that the demand shocks they consider–consumption preference or money supply–do not affect *efficient* level of market tightness. Under competitive search, tightness is necessarily at the efficient level, so some deviation would thus be necessary for such demand shocks to matter.

However, under the matching setup considered here, the efficient level of market tightness also depends on labor hours and technology. It follows that any demand shock that affects labor demand also raises D and the Solow residual. In the current bare-bones setup, a reduction in θ_n stimulates labor demand, which raises shopping and tightness. Additionally, we included money as Michaillat and Saez (2015), then a consumption preference shock or shock to the level of money supply would also affect labor and hence tightness.

In general, the influence of labor hours on the efficient level of tightness holds provided that the expenditure ρ does not scale one-for-one with capacity. If the cost of a shopping were ρF instead of ρ, then we would instead have $T = \rho/(\Psi_d - \rho)$ and D would be determined by $\rho = \phi\Psi_d$. The efficient level of tightness would just depend on ϕ, A, and ρ. We believe it plausible a priori that shopping expenditure costs scale less than one-for-one with firm capacity, though of course parsing these micro-level features require more granular data and research.