Unemployment and Labor Productivity Comovement: the Role of Firm Exit

Miroslav Gabrovski[b], Mario Rafael Silva[a]

[a] *Hong Kong Baptist University*
[b] *University of Hawai'i Manoa*

Abstract

The Diamond-Mortensen-Pissarides model implies a nearly perfect correlation between labor productivity and unemployment, yet the empirical relationship is mild. We show that incorporating sunk entry costs and a congestion channel of vacancy creation in an otherwise standard setup can reconcile the discrepancy. Sunk costs cause vacancies to be a positively valued, predetermined variable. If the destruction shock is infrequent, most vacancies were created in the past, so the number of vacancies correlates more closely with past than contemporaneous productivity. The model, calibrated to match micro-level evidence on product and firm destruction, matches both the contemporaneous and dynamic correlations between productivity and unemployment.

JEL Classification: E24, E32, J63, J64

Key words: job destruction; entry costs; unemployment; aggregate fluctuations; dynamic correlations

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1. Introduction

The Diamond-Mortensen-Pissarides model (DMP henceforth) has been the primary workhorse for studying the business cycle properties of unemployment, labor market tightness, and vacancies. Following Shimer (2005), much of the literature has focused on reproducing the empirically observed volatilities of these three labor market variables.[2] The majority of these past studies follow the tradition of using technology shocks the real business cycle literature and utilize technology shocks as the fundamental driving force behind business cycles. As a consequence, the model predicts near-perfect cross-correlation between productivity and the labor market variables. This result contrasts sharply with the data, in which the correlation is only mild.

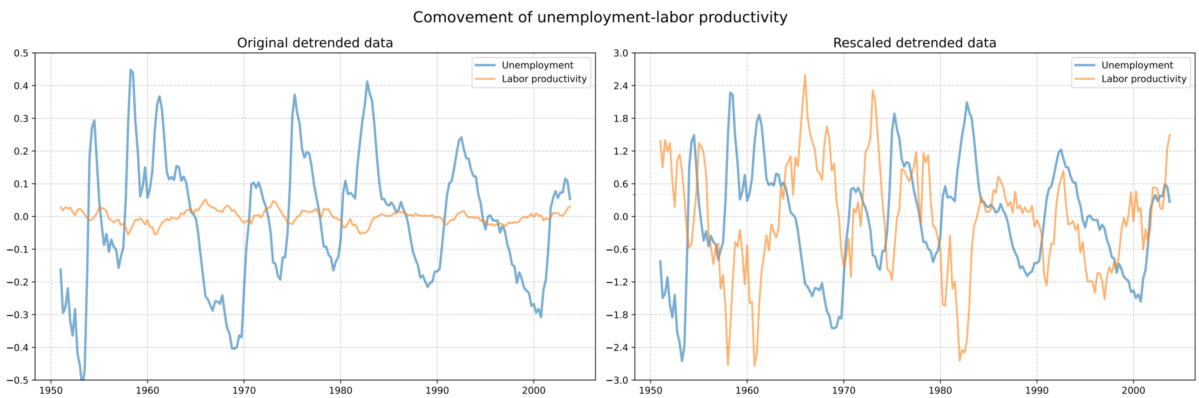

Figure 1: Quarterly unemployment and labor productivity. Unemployment is measured in number of persons as Shimer (2005) and productivity is measured in terms of output per worker. Each series is logged and detrended using an HP filter with smoothing parameter of 10^5. In the right panel, each series is also rescaled to have a standard deviation of unity.

Figure 1 plots HP-filtered quarterly unemployment and labor productivity between 1951 and 2003. We chose this time length to be consistent with Shimer (2005). The two series exhibit moderate negative comovement overall, and the magnitude of the correlation is more pronounced earlier in the sample. Indeed, the correlation between the two series is -0.43. Detrending the series using the Hamilton regression filter instead with a forecast horizon of

[2] Some notable examples include Hagedorn and Manovskii (2008), Hall and Milgrom (2008), Pissarides (2009), Ljungqvist and Sargent (2017).

2 years and using the 4 most available series, the correlation is -0.34.

Mortensen and Nagypal (2007) argue that this discrepancy in the implied correlation between model and data points to an important driving force of business cycle dynamics absent in the baseline DMP framework. Following this line of thought, Barnichon (2010) develops a model with both technology and demand shocks that can reproduce a mildly countercyclical unemployment rate. Similarly, Gervais et al. (2015) incorporate a proficiency ladder in the DMP model and show that shocks to the ease of learning-by-doing can generate an unemployment-productivity correlation close to that the data. Coles and Moghaddasi Kelishomi (2018) build a model with both technology and separation shocks that breaks the near-perfect correlation between productivity and the labor market variables. In contrast to this existing literature, we propose a mechanism which endogenously reduces the magnitude of the correlation between the labor market variables and productivity and relies only on a single shock to technology.

In particular, augmenting the DMP model with sunk vacancy creation costs and carefully distinguishing between the destruction rates of a match and a product line can reconcile the model-implied cross-correlations with those observed in the data. Sunk costs render vacancies a positively valued asset in equilibrium. Furthermore, the stock of vacancies is predetermined and influenced by the likelihood of destruction or obsolescence shocks, as well as expectations of future profits.

When the destruction rate is high, vacancies have shorter lifespans, resulting in a vacancy pool predominantly composed of newly created job opportunities. Consequently, vacancies—and by extension, market tightness and unemployment—are highly correlated with current productivity, which drives expectations for future profits. Conversely, if the shock occurs less frequently, most current vacancies are derived from past creations. In this scenario, labor market variables tend to co-move more closely with past productivity than with the current state of technology.

Our numerical exercises indicate that when the destruction rate is calibrated to align with (i) micro-level evidence on product destruction and firm exits or (ii) values commonly used in growth literature, the model successfully reproduces the empirically observed mild correlation between productivity and labor market variables while still maintaining a strong cross-correlation among those labor market variables.

Following Diamond (1982), a small but growing literature highlights the importance of sunk vacancy creation costs in the context of the DMP model for explaining labor market dynamics. Notable examples include Fujita and Ramey (2007), who focus on the sluggish response of the market tightness to productivity shocks; Shao and Silos (2013), who stress the dynamics of the value of a vacancy; Coles and Moghaddasi Kelishomi (2018), who emphasize the drivers of unemployment volatility over the cycle; Mercan and Schoefer (2020), who concentrate on replacement hiring; Haefke and Reiter (2020), who examine match cyclicality and wages; Potter (2022), who focuses on modern search technologies, and Qiu (2023), who analyzes vacancy dynamics and the decision of whether or not to participate in the labor force.

We contribute to this literature by highlighting the importance of sunk vacancy creation costs for another aspect of labor market dynamics — the comovement between labor productivity and labor market variables (unemployment, vacancies, market tightness). Our model environment nests Coles and Moghaddasi Kelishomi (2018) as a special case in which all worker-firm pair dissolutions are due to job destruction, i.e. there are no quits. Specifically, creating a new vacancy requires an up-front investment in a new technology. This sunk investment is drawn randomly from a known exogenous distribution.[3] Consequently, each vacancy carries a positive asset value. As a result, when a worker quits, the firm strictly prefers to keep its vacancy open.

We distinguish this separation shock from a destruction shock, in which the vacancy is lost alongside the job. Such a disturbance makes the firm's product obsolete or destroys its business opportunity. That is, if the destruction shock hits, the firm exits the labor market altogether. The environment in Coles and Moghaddasi Kelishomi (2018) is a special case

[3]The current setup with a distribution of investment costs is isomorphic to one in which there is a congestion externality in entry. That is, the entry cost each firm has to pay in order to open a vacancy is increasing in the number of entrants. As Fujita and Ramey (2007) show congestion in entry leads firms to spread out their response to productivity shocks over several periods. This gradual propagation of shocks also serves to reduce the contemporaneous correlation between productivity and the labor market variables. However, this property on its own is not enough to reproduce the mild correlations observed in the data.

of ours in which the separation rate is zero. Our environment also nests that of Fujita and Ramey (2007) as a special case, since we use a more flexible sunk investment cost function. Compared to the model in our paper, the one by Shao and Silos (2013) differs mostly from the role of capital in their economy as the driving source of congestion in vacancy creation — the more firms enter, the higher the demand for capital which increases its rental price and, as a consequence, the equilibrium cost of vacancy creation. The model in Mercan and Schoefer (2020) incorporates job-to-job flows, which we abstract from.

A negative productivity shock lowers future expected profits, which dampens vacancy creation. In the standard DMP setup vacancies fall instantaneously, which causes labor market variables to comove almost perfectly with productivity. In the current setting, however, vacancies are long-lived assets with a positive value — a stock variable. Hence, the number of vacancies in the market is correlated not only with the current technology shock, but also with past ones that may have (dis)incentivized entry in previous periods. As the expected life of a vacancy rises, so does the history of past shocks that affects the pool of vacancies today. This entails a larger fraction of vacancies created in the past and thus a lower correlation between vacancies and current productivity. Consequently, the magnitude of the cross-correlations between labor market variables and productivity in the model depends on how long-lived vacancies are, i.e. the size of the destruction shock.

Consequently, disciplining the size of the destruction shock plays a central role. Initial vacancy creation requires an up-front sunk investment on the part of the firm in order to capture a business opportunity. As the destruction shock most closely resembles the loss of a business opportunity, it is appropriate to calibrate it using data on product obsolescence or firm exit. The separation shock, in turn, can be imputed using data on job loss and separations. Furthermore, there is empirical evidence that product turnover and firm exit are tightly linked to employment turnover, e.g. Bernard et al. (2010), Lee and Mukoyama (2015). Intuitively, when firms retire production lines this causes an organizational change, which induces a shift in the labor needs of the company. This change likely both reshuffles workers to different roles within the company and destroys jobs and vacancies. Using micro-level evidence Broda and Weinstein (2010) find an annual destruction rate of 3%. This value is consistent with the obsolescence rate of 3% that Comin and Gertler (2006) calibrate using balanced growth path restrictions from the U.S. data. This value is also within the ball park of the 5% − 6% annual destruction rate implied by the estimates in Bernard et al. (2010).[4] Alternatively one may turn to the data on firm exit, which the growth literature has commonly used.[5] In particular, Broda and Weinstein (2010) find firm exit to be about 10% annually. This figure also accords with the estimates of job destruction in Lee and Mukoyama (2015), which the authors obtain from analyzing data on plant-level entry and exit over the business cycle. Since destruction is likely to be caused by both firm exit and product turnover we take the mid point of the estimates and calibrate our benchmark to be consistent with the 6% annual destruction that Bernard et al. (2010) find in the data. We also perform a robustness check utilizing the 10% rate implied by the evidence on firm exit alone.

In both cases the model reproduces the mild correlation between productivity and all three of the labor market variables of interest (vacancies, the market tightness, and unemployment) reasonably well. Moreover, the cross-correlations between the labor market variables themselves remain strong. We highlight how our calibration strategy of the destruction rate complements and improves on several shortcomings in the existing literature in section 3.1. In addition, we show that matching the empirical correlation between unemployment and productivity does not have to come at the expense of the model's ability to match the relative volatility of unemployment with respect to productivity. A calibration of our model along the lines of Hagedorn and Manovskii (2008) is able to reproduce both the empirically observed correlation between the two series and the relative standard deviation of unemployment.

Related Literature. Although the literature has mainly focused on the unemployment volatility puzzle, several studies have also stressed the correlation between labor productivity and unemployment. Notably, Barnichon (2010) highlights the stylized fact that these two series are only mildly correlated, using a variety of productivity measures. Furthermore, he finds the cross-correlation to be negative pre-1984 and positive post-1984, though it is mild in

[4]In particular, the authors estimate that between 26% and 29% of firm output is accounted for by products about to be dropped in the next five years.

[5]See, for example Bilbiie et al. (2012) and Gabrovski (2019).

both periods. Barnichon (2010) explains the empirically observed sign change of the correlation using demand and supply shocks and nominal rigidities. In related work Hagedorn and Manovskii (2011) examine some of the empirical shortcomings of the DMP model, including the discrepancy between the theoretically predicted cross-correlation between unemployment and productivity and its empirical counterpart. The authors reproduce the mild magnitude of the correlation by incorporating a stochastic home production value and a time-to-build lag for vacancies. Intuitively, both features reduce the strength of the correlation between productivity and unemployment almost mechanically. First, wages, and consequently the firm's surplus from the match, depend on home production, so the model features a second independent source of volatility that affects firms' vacancy posting decisions. Second, because of the time-to-build lag, vacancies which enter the labor market today are not correlated with current productivity shocks. Furthermore, the time-to-build assumption allows the authors to match a qualitative feature of the dynamic correlations in the data — the peak correlation between vacancies and productivity occurs when productivity is lagged two quarters. In contrast to both these studies, our model features a technology shock as the single source of exogenous volatility in the model. Moreover, we can match the low correlation between unemployment and productivity and approximately fit the dynamic correlations without targeting any of these features in the calibration.

Gervais et al. (2015) develop a model with learning-by-doing that is able to reproduce the empirically observed cross-correlation using a single source of exogenous uncertainty. They examine shocks to the rate with which workers learn on the job in lieu of technology disturbances. Given a positive learning shock, firms increase hiring because they expect high future profits, which reduces unemployment. The effect of the shock on productivity, however, is indirect and only works through a labor force composition effect. A higher rate of learning makes it easier for workers to increase their proficiency, which raises aggregate productivity subject to a lag. This breaks the immediate response of productivity in the standard DMP model and delivers a lower magnitude of the cross-correlation between productivity and unemployment. Compared to this study, our setup can match the data without departing from standard technology shocks as the driving force behind business cycle fluctuations.

2. Model

2.1. Environment

We closely follow a conventional equilibrium unemployment model in discrete time, e.g. Pissarides (2000). The only point of departure is in the vacancy entry mechanism — we assume sunk vacancy creation costs along side a finitely elastic vacancy creation, i.e. there is congestion in vacancy creation. Unemployed workers search for jobs, firms search for workers to fill their vacancies, and matches are formed according to a matching function. Once matched, workers and firms decide on wages using Nash Bargaining and the match persists until the pair exogenously separates. There is a fixed measure $F > 0$ of firms that can create vacancies. In each period firms receive access to a new independent business opportunity, which can be undertaken by paying an investment cost x. This cost reflects, for example, the costs associated with R&D and taking a new product to the production phase. Let Q_t denote the value of posting a vacancy at time t, and suppose each firm draws an investment cost from a known distribution G. Given that firms undertake their business opportunity if and only if $x \leq Q_t$, the aggregate amount of new vacancy creation is $e_t = FG(Q_t)$. Appendix B shows that the model is isomorphic to having a constant sunk entry cost that depends positively on the number of entrants.[6]

Each period a number $M(u_t, v_t)$ of firm-worker pairs are formed, where u_t denotes unemployment and v_t the number of vacancies on the market. As is standard, the matching function is increasing and concave in each of its arguments and exhibits constant returns to scale. Thus, the job-filling rate for firms is $q(\theta_t) \equiv M(u_t, v_t)/v_t = M(\theta_t^{-1}, 1)$ and the job-finding rate is $f(\theta_t) \equiv M(u_t, v_t)/u_t = M(1, \theta_t)$. With probability s, a worker separates from the job. In this event, the firm keeps its business opportunity, but must hire a new worker for the job. Moreover, firms and matches are subject to a destruction shock–with probability δ the firm's business opportunity becomes obsolete. The match is dissolved, and

[6] Note that our entry condition implies there is congestion in vacancy creation — the more firms post vacancies each period, the higher the average investment firms must make. We can arrive at an analogous congestion mechanism if we instead assume that firms compete for new business opportunities that come in the form of innovations. See, for example, Gabrovski (2019) and Gabrovski (2022).

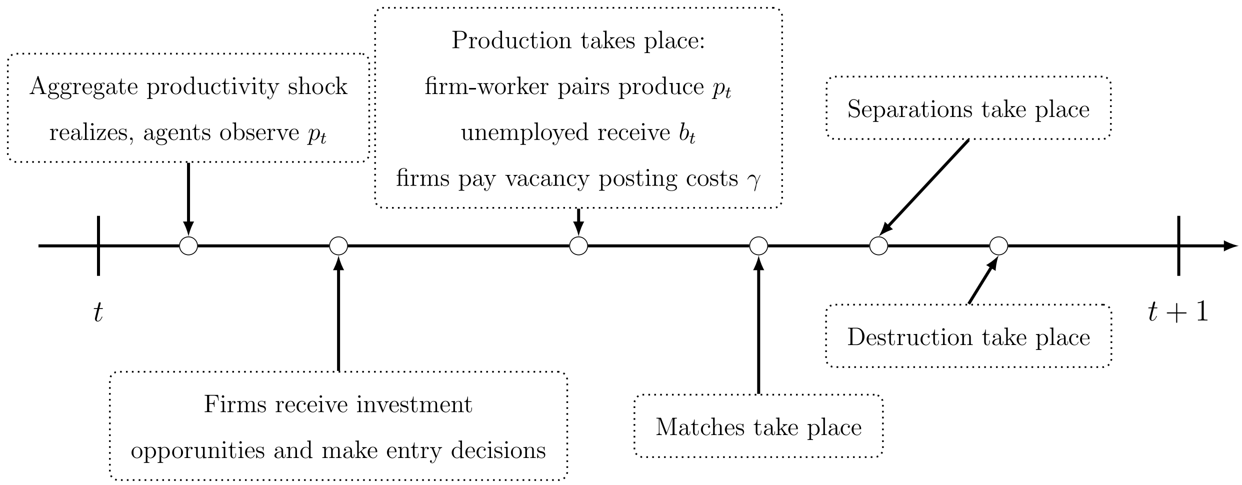

Figure 2: Labor Market Timing

so is any unfilled vacancy.

Figure 2 summarizes the timing in our model. At the beginning of each period the aggregate productivity shock is realized and agents observe the current productivity level p_t. Next, firms receive their investment opportunities and make entry decisions. Third, production takes place: firm-worker pairs that are matched produce p_t, workers are paid a wage w_t, and unemployed workers receive benefits b_t. Furthermore, at this stage firms which have an unfilled vacancy pay the vacancy posting cost γ. Fourth, matching takes place. Fifth, worker separation and firm destruction take place. A match formed in period t is not subject to a separation shock at time t, but may be hit with a destruction shock. Moreover, newly created vacancies can also be destroyed.

Our environment nests Coles and Moghaddasi Kelishomi (2018), where all job loss is due to destruction, i.e. $s = 0$. In the event of a separation shock, the firm keeps its vacancy and begins searching for a worker right away. In the event of a destruction shock, however, the firm has to make an entry decision. As we show in the next section, the quantitative predictions of the model depend greatly on the relative importance of these two forces. If the destruction shock dominates, then dynamics are close to that of the baseline Pissarides (2000) model and vacancy creation is mainly determined by the current labor market conditions; otherwise, the separation shock dominates and the dynamics are very different because the mass of vacancies is mainly determined by *past* labor market conditions.

2.2. Bellman equations

The value of a vacancy, Q_t, comprises several terms. First, firms must pay a vacancy posting cost γ in order to search for workers in the labor market. If they are matched with a worker, which occurs with probability $q(\theta_t)$, their vacancy transitions to a filled job. Otherwise, they keep the opportunity to search for a worker next period. Finally, firms discount the future with a factor β and expect their business opportunity to remain viable $(1-\delta)$. Letting J_t denote the value of a filled job, then the value of a vacancy Q_t satisfies

$$Q_t = -\gamma + \beta(1-\delta)\mathbb{E}_t\left[q(\theta_t)J_{t+1} + (1-q(\theta_t))Q_{t+1}\right]. \tag{1}$$

A filled job has productivity p_t. Firms pay workers a wage w_t, so the per-period profits are $p_t - w_t$. With probability $(1-s)(1-\delta)$ the firm and the worker do not separate and the business opportunity does not become obsolete, so the firm keeps its filled job next period. There is a chance $s(1-\delta)$ that the firm-worker pair dissolves due to separation (rather than destruction). In that event, the firm keeps a vacancy with expected value $\mathbb{E}_t Q_{t+1}$ and can search for a new worker next period. Lastly, in the event that the business opportunity is destroyed, the firm-worker pair dissolves and the firm exits the market. Thus,

$$J_t = p_t - w_t + \beta\left[(1-s)(1-\delta)\mathbb{E}_t J_{t+1} + s(1-\delta)\mathbb{E}_t Q_{t+1}\right]. \tag{2}$$

An unemployed worker receives benefits b. With probability $f(\theta_t)$ she is matched with a firm and, conditional on that, there is a $(1-\delta)$ chance the job survives until the beginning of next period. In that event the worker becomes employed. Otherwise, the worker remains unemployed. Thus, the value of unemployment, U_t, satisfies

$$U_t = b + \beta\left[(1-\delta)f(\theta_t)\mathbb{E}_t W_{t+1} + [1-(1-\delta)f(\theta_t)]\mathbb{E}_t U_{t+1}\right]. \tag{3}$$

An employed worker receives wages w_t. She keeps the job whenever the firm-worker pair do not separate and the firm survives the destruction shock, i.e. with probability $(1-\delta)(1-s)$. Otherwise, the worker loses the job and transitions to unemployment next period. Let $\tau = 1-(1-\delta)(1-s)$ denote the aggregate separation rate. The value of the job for the worker, W_t, satisfies

$$W_t = w_t + \beta\left[(1-\tau)\mathbb{E}_t W_{t+1} + \tau\mathbb{E}_t U_{t+1}\right]. \tag{4}$$

2.3. Wages, laws of motion, and entry

As is standard in the literature, wages are determined according to Nash bargaining:

$$w_t = \arg\max_{w_t} [J_t - Q_t]^{1-\alpha}[W_t - U_t]^{\alpha}, \tag{5}$$

where α is the worker's bargaining power. Under linearity of the value functions, (5) yields a fixed share of the surplus to each party:

$$\alpha(J_t - Q_t) = (1-\alpha)(W_t - U_t). \tag{6}$$

Lastly, we close the model by specifying the laws of motion and the entry decision of firms. As entry into the labor market requires firms to make a sunk investment cost, vacancies are a state variable. In particular, the number of vacancies today v_t is the sum of three terms. First, all vacancies last period that were not matched with a worker and survived the destruction shock, $(1-\delta)[1-q(\theta_{t-1})]v_{t-1}$, remain in the pool of vacancies today. Second, all filled jobs that experienced a separation shock, but survived the destruction shock last period, $(1-\delta)s(1-u_{t-1})$, transform into vacancies today. Lastly, all new entrants, e_t, post a vacancy. Hence,

$$v_t = (1-\delta)[(1-q(\theta_{t-1}))v_{t-1} + s(1-u_{t-1})] + e_t. \tag{7}$$

The amount of new entrants satisfies the free entry condition $e_t = FG(Q_t)$. We follow Coles and Moghaddasi Kelishomi (2018) and postulate a parsimonious power law distribution: $G(Q_t) = Q_t^{\xi}$.[7] Note that under this specification the value of the vacancy in equilibrium is $Q_t = (e_t/F)^{1/\xi}$. Thus, the elasticity of the value of a vacancy with respect to entry is $1/\xi$. Conversely, the elasticity of entrants to the value of a vacancy, ξ, is positive and finite. In contrast, under the baseline DMP model with no congestion this elasticity is infinite.

Due to the destruction shock, the law of motion for unemployment differs from the one in the Pissarides model as well. In particular, worker-firm pairs are subject to a destruction shock immediately after the match, before production takes place. Thus, only a fraction $(1-\delta)f(\theta_{t-1})$ of unemployed workers from the previous period transition to employment this period. The rest remain unemployed. Therefore,

$$u_t = [1-(1-\delta)f(\theta_{t-1})]u_{t-1} + \tau(1-u_{t-1}). \tag{8}$$

[7] Beaudry et al. (2018) and Potter (2022) use similar functional forms.

2.4. Exogenous productivity process

The natural logarithm of productivity follows an AR(1) process

$$\log(p_t) = \rho \log(p_{t-1}) + \epsilon_t, \tag{9}$$

where ρ is a persistence parameter, and $\epsilon_t \sim N(0, \sigma)$ is white noise.

2.5. The job creation condition

Combining the free entry condition, $Q_t = (e_t/F)^{1/\xi}$, and the Bellman equations for vacancies (1) and for filled jobs (2) yields the job creation condition in our economy:

$$\frac{\gamma + K_t}{q(\theta_t)} = \beta(1-\delta)\mathbb{E}_t\left[p_{t+1} - w_{t+1} - K_{t+1} + (1-s)\left(\frac{\gamma + K_{t+1}}{q(\theta_{t+1})}\right)\right] \tag{10}$$

where $K_t \equiv \mathbb{E}_t(Q_t - \beta(1-\delta)Q_{t+1}) = \mathbb{E}_t\left[e_t^{1/\xi} - \beta(1-\delta)e_{t+1}^{1/\xi}\right]/F^{1/\xi}$ is the expected flow entry cost: the difference between the entry cost the firms face today and the discounted expected entry cost tomorrow. The interpretation of (10) is analogous to that in the baseline DMP model: the left-hand side of the equation represents the expected costs of posting and maintaining a vacancy, whereas the right-hand side is the expected profit.

The costs in our environment account for the role of congestion. For example, if many firms are entering the market today then K_t rises and an entrant may choose to delay entry. This force is captured by a higher K_t in the job creation condition above. This smoothing mechanism yields a hump-shaped response in vacancies, a feature consistent with the empirical evidence provided by Fujita and Ramey (2007). In contrast, vacancies peak on impact in the standard DMP model.[8] The expected benefit from posting a vacancy is the expected discounted profits next period, $p_{t+1} - w_{t+1}$, plus the continuation value of the vacancy in the event the firm and worker separate, $(1-s)[(\gamma + K_{t+1})/q(\theta_{t+1})]$, net of the expected flow costs, K_{t+1}.

We next characterize wages according to Nash bargaining. Using the surplus sharing rule (6) together with the four Bellman equations (1), (2), (3), and (4) yields an expression of the

[8] Fujita and Ramey (2007) study this mechanism in detail and show that it helps the model match key properties of vacancies in the data. Unlike us, however, Fujita and Ramey (2007) do not focus on the mild procyclicality of vacancies. Indeed in their model vacancies and productivity are highly correlated.

wage that generalizes the one in the baseline DMP model:

$$w_t = \alpha \left[p_t - K_t + \frac{\theta_t}{1-\delta}(\gamma + K_t) \right] + (1-\alpha)b \tag{11}$$

The wage is a weighted average between the flow payoff from the match and the worker's outside option of receiving unemployment benefits. The benefit of the match constitutes the match output together with the search costs the firm saves from not posting a vacancy, $p_t + \frac{\theta_t}{1-\delta}(\gamma + K_t)$.

The only difference of (11) from the baseline model is that now the job opportunity has a finite expected life of only $1/(1-\delta)$ and that the vacancy costs include both γ and the flow entry cost K_t. Additionally, vacancies are an asset with a positive value and are subject to congestion in our economy. Consequently, the wage includes an additional term: $-K_t$. This term reflects the fact that the outside option of firms in the bargaining game has a flow value K_t. If vacancies are expected to be more valuable tomorrow, then the firm will have an easier time recruiting a worker tomorrow, which raises the attractiveness of her outside option and subsequently reduces the wage. Alternatively, if the expected value of a vacancy tomorrow is lower the firm is more eager to match with a worker today and is thus willing to offer a higher wage.

2.6. Equilibrium

We have the necessary ingredients to define equilibrium.

Definition 1. *An equilibrium is an infinite, bounded sequence of productivity, wages, market tightness, entrants, vacancies, and unemployment* $\{p_t, w_t, \theta_t, e_t, v_t, u_t\}_{t=0}^{\infty}$ *such that, given initial conditions* (u_0, v_0, p_0)*, (i) firms set entry optimally according to* (10)*; (ii) the wage solves the Nash Bargaining problem between the firm and the worker as in* (11)*; (iii) vacancies follow the low of motion* (7)*; (iv) unemployment follows the law of motion* (8)*; (v) productivity follows the AR(1) process defined in* (9)*.*

Before we turn to the quantitative analysis, we summarize the steady state of the model. Let $r = 1/\beta - 1$ denote the rate of time preference.

Definition 2. *A steady-state equilibrium is a list* (K, e, w, θ, u, v) *such that*

$$H = \frac{\gamma + K}{q(\theta)} = \frac{1-\delta}{r+\tau}(p - w - K) \tag{12}$$

$$K = \left(\frac{e}{F}\right)^{1/\xi} \frac{r+\delta}{1+r} \tag{13}$$

$$w = \alpha[p - K + \frac{f(\theta)}{1-\delta}H] + (1-\alpha)b \tag{14}$$

$$u = \frac{\tau}{\tau + (1-\delta)f(\theta)} \tag{15}$$

$$e = \delta(v + 1 - u) \tag{16}$$

$$\theta = \frac{v}{u}$$

We note from (12) that the flow sunk entry cost K affects both the profit flow and hiring cost. Equation (13) makes it transparent that the elasticity of flow entry costs with respect to entrants is $1/\xi$. Finally, (16) highlights that entrants replace a fraction δ of vacancies and employed. Thus, the destruction rate directly regulates the relative size of entrants among vacancies.

There is generally at most one steady state, and existence depends on certain parametric conditions:

Proposition 1. *There is at most one steady state. The steady state exists if and only if*

$$\left(\frac{\delta}{1-\delta}\frac{\tau+1-\delta}{F}\right)^{1/\xi} \frac{r+\delta}{1+r} < \frac{(1-\delta)(1-\alpha)(p-b) - \gamma(r+\tau)}{r+\tau+(1-\delta)(1-\alpha)}.$$

Existence is more likely to hold with higher productivity relative to unemployment value $p-b$, lower rate of time preference r, a lower destruction rate δ, and a higher mass of potential firms F.

We can also characterize the steady state entirely in terms of market tightness. Combine (12) and (14) to express the steady-state job creation condition as

$$\frac{\gamma + K}{q(\theta)} = \frac{(1-\delta)(1-\alpha)(p-b-K)}{r+\tau+\alpha f(\theta)}$$

and rearrange as

$$\frac{\gamma + K}{q(\theta)} + \frac{(1-\delta)(1-\alpha)K}{r+\tau+\alpha f(\theta)} = \frac{(1-\delta)(1-\alpha)(p-b)}{r+\tau+\alpha f(\theta)}.$$

Note that by letting $K \to 0$ and $\delta \to 0$, $\tau \to s$, and the steady-state job creation condition coincides with that of the standard model:

$$\frac{\gamma}{q(\theta)} = \frac{(1-\alpha)(p-b) - \gamma\alpha\theta}{r+s}$$

3. Quantitative Analysis

The appendix describes the global numerical procedure we use to simulate the model and to derive the impulse response functions, which involves approximating the policy functions with polynomials and iterating on the Euler equations until finding a fixed point of the coefficients. In this section, we outline the calibration and results, and focus on the underlying mechanism. Namely, we show that the model-implied cross-correlation between productivity and labor market variables depends crucially on the calibrated value of the destruction shock. Nevertheless, the labor market variables themselves remain nearly perfectly correlated with each other for any value of δ.

Intuitively, the model can reproduce the mild cross-correlation for two reasons. First, vacancies are a predetermined variable, which evolve according to the law of motion (7). As is evident in the equation, the pool of vacancies comprises new entrants and surviving vacancies created in previous periods. This feature contrasts with the standard DMP model in which entrants constitute all vacancies. As a result, in the baseline DMP model vacancies are a choice variable and adjust instantaneously in response to changes in labor productivity. In contrast, when there is a fixed cost of entry, only a fraction of the vacancy pool, e_t, is determined by current labor market conditions whereas the remaining fraction of vacancies, $(1-\delta)[(1-q(\theta_{t-1}))v_{t-1} + s(1-u_{t-1})]$, is correlated with past labor productivities. When the destruction shock δ is relatively high, the contribution of entrants to the mass of vacancies is relatively high, so the correlation between vacancies and labor productivity is relatively high as well. On the other hand, if the shock is less frequent, most vacancies from the previous period survive, so that only a small fraction of current vacancies constitute new entrants. Second, the economy features a congestion channel: the average cost of entry rises with the number of entrants. This feature generates a smoothing mechanism, which induces firms to smooth out their response to productivity shocks over time to mitigate high entry costs.

Fujita and Ramey (2007) highlight this mechanism and show it reproduces the hump-shaped response of vacancies to productivity shocks that we observe in the data.

3.1. Calibration

Our paper highlights how sunk investment in vacancy creation affects the DMP model's ability to reproduce the mild cross-correlation between productivity and labor market variables. In this theoretical framework, creating a vacancy is intrinsically linked to a business opportunity. Job loss in the economy results from two distinct processes: (i) a separation between firm and worker that dissolves the match but preserves the firm's business opportunity, and (ii) destruction of the business opportunity that renders the firm's product no longer viable, forcing the firm to exit the labor market. The first source aligns with the standard DMP model's view of separations: matches dissolve when workers relocate, change jobs due to management conflicts, are terminated for cause, etc. The second source relates to the firm's business environment: matches end when competitors capture market share, products become obsolete, or firms cease operations. Therefore, we calibrate the destruction shock specifically to capture the firm's lost business opportunity.

To facilitate comparison with existing literature, we maintain a calibration closely aligned with Coles and Moghaddasi Kelishomi (2018). Our only departure is in calibrating the destruction and separation shocks. In the benchmark, we use data on product destruction and firm exit to calibrate δ. We then present the model's implied moments for various values of δ used in literature to highlight how numerical results depend on the calibrated destruction shock value.

We use the matching function introduced in Den Haan et al. (2000): $M(u, v) = uv/(u^\nu + v^\nu)^{1/\nu}$ for $\nu > 0$. This form bounds matching probabilities between 0 and 1:

$$f(\theta) = (1 + \theta^{-\nu})^{-1/\nu}, \quad q(\theta) = (1 + \theta^{\nu})^{-1/\nu}$$

The model frequency is monthly. We set $\delta = 0.0051$ to match a 6% annual destruction rate following the evidence in Bernard et al. (2010). This value is also in the mid point between the evidence on product destruction and firm exit in Broda and Weinstein (2010). Existing studies within the labor search literature have targeted job destruction rates in alternative ways. For example, Coles and Moghaddasi Kelishomi (2018) set the separation rate to zero and attribute all job separations in the data to destruction. Fujita and Ramey (2007), on the other hand, identify the destruction and separation rates using evidence on total job losses from the Business Employment Dynamics (BED) program, coupled with the usual moments of the job-finding rate and steady state unemployment. A similar strategy is used by Shao and Silos (2013) who pin down the destruction rate using evidence on total job losses from Shimer (2005) and a moment restriction for the steady state level of unemployment. Mercan and Schoefer (2020) use survey evidence from German employers to distinguish between hiring aimed at replacing workers who have quit (replacement hiring) and new job creation to separately calibrate their destruction and separation rates.

Our calibration offers a viable alternative that addresses several shortcomings in previous literature. We follow Coles (2018) in our theoretical framework, allowing vacancy creation congestion to follow a flexible functional form calibrated with empirical evidence. With this additional degree of freedom, the model can simultaneously match the aggregate job loss rate, unemployment, and job finding rate in steady state without restricting the destruction rate, unlike Shao (2013). Compared to Coles (2018), our calibration strategy leverages evidence on product/firm destruction, eliminating the need to attribute all job separations to destruction.[9] Unlike Mercan and Schoefer (2020), our approach relies exclusively on U.S. data. While they accurately derive the destruction rate from data on new jobs versus re-hires, their German data presents a limitation for our purposes, as German and U.S. labor markets likely differ in key aspects affecting destruction and separation rates and their business cycle properties. Furthermore, we utilize only moments readily available from public data. Fujita and Ramey (2007) use job destruction evidence from the BED program reported in Faberman et al. (2004) for their calibration. Since job destruction is defined as "the gross number of jobs lost at establishments either closing down or contracting their workforce" (Faberman, 2004, p.1), identifying the destruction rate in their model requires deriving it from the equilibrium using job separation and finding rates. In contrast, our calibration strategy identifies the destruction rate directly using job destruction evidence alone.

[9] To be precise, Coles and Moghaddasi Kelishomi (2018) specify their model so that all separations lead to the loss of a vacancy. Thus, in contrast to their framework we make the distinction between separations and destruction.

The discount factor $\beta = 0.9975$ is chosen so that it matches an annual discount rate of 3%. Further, we set the vacancy posting costs γ to zero, following Coles and Moghaddasi Kelishomi (2018) and set $b = 0.9$.[10]. ξ is set to 1, so that the distribution $H(Q_t)$ is uniform, following Coles and Moghaddasi Kelishomi (2018). The parameter α is chosen so that the approximate steady-state elasticity of wages to productivity, $\alpha p/w$, equals 0.6.

To match the empirical job-filling and job-finding rates we set $1/[(1-\delta)q(\theta)] = 0.75$ months and $1/[(1-\delta)f(\theta)] = 2.2$ months. Further, the mean job separation probability is set to $\tau = 3.4$ percent per month, thus steady state unemployment is $u = 6.95\%$. Given δ and ξ these three moments jointly yield $\nu = 1.59$, $s = 0.029$, and $F = 0.000699$. The only source of uncertainty in our model is the productivity shock. We fix the AR(1) autoregression coefficient $\rho = 0.979$ and standard deviation $\sigma = 0.007$, similar to the values used by Coles and Moghaddasi Kelishomi (2018). Table 1 below summarizes the calibration.

Preferences/Technology	Parameter	Value	Calibration Strategy
Vacancy posting cost	γ	0	Coles and Moghaddasi Kelishomi (2018)
Bargaining power	α	0.566	Elasticity of wages to productivity
Unemployment benefits	b	0.9	Fixed
Matching function elasticity	ν	1.59	Job-finding rate
Discount factor	β	0.9975	3% Annual discount rate
Separation rate	s	0.029	3.4% Monthly match dissolution probability
Destruction rate	δ	0.0051	6% annual destruction rate
Population of firms	F	0.000699	Job-filling rate
Cost distribution parameter	ξ	1	Coles and Moghaddasi Kelishomi (2018)

Table 1: Calibration

3.2. Impulse Response Functions and Mechanism

To highlight the intuition behind our numerical results we first turn to the impulse response functions generated by the model. To this end we graph the economy's response to a

[10]This value is in between 0.71 from Hall and Milgrom (2008) and 0.95 from Hagedorn and Manovskii (2008)

one standard deviation negative technology shock in Figure 3. On impact vacancies, entry, and the market tightness all decrease. Entry initially responds the most and only slowly recovers in subsequent periods. This sluggish response is due to the congestion in the model: increasing entry faster leads to larger vacancy creation costs, which incentivizes firms to delay entry to future periods. Since vacancies are an asset with a positive value firms do not voluntarily exit the labor market. Instead, they maintain their vacancies, so the only change in the pool comes from the reduced entry. Thus, vacancies respond very little on impact and achieve their maximum response only around 3 years after the initial shock. Market tightness and unemployment follow a similar pattern because their behavior is a direct consequence of firm's entry decisions. As highlighted by Fujita and Ramey (2007) this sluggishness (i) is absent from the baseline DMP model without congestion in entry and (ii) is well-supported empirically. In what follows we highlight the importance of this sluggishness for the model's ability to match the cross-correlation between productivity and the labor market variables observed in the data.

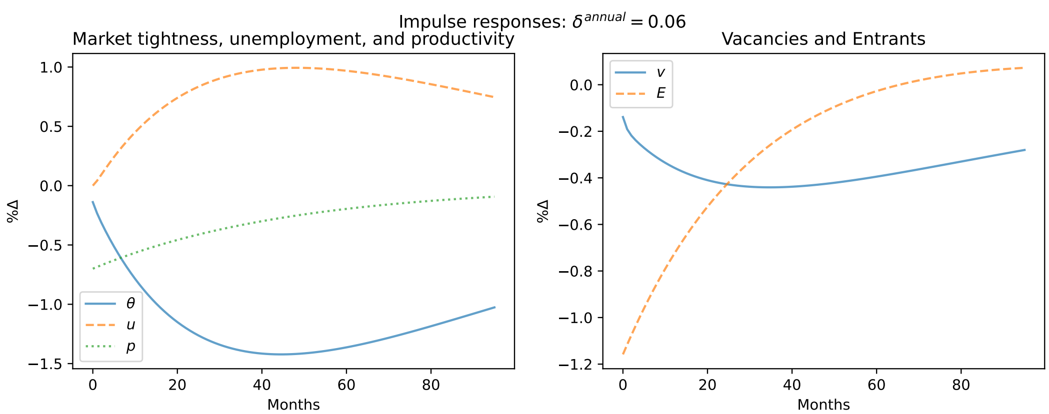

Figure 3: Impulse response functions to a one standard deviation negative technology shock in the benchmark calibration with $\delta = 0.0051$ (6% annual)

Next, we focus on the impulse response functions generated by the model when it is calibrated to match the data on firm destruction. The destruction shock is set to $\delta = 0.00874$ so that the annual rate of firm exit is 10% following the evidence in Broda and Weinstein (2010). We recalibrate the other parameters to match the targets in Table 1. Figure 4 presents the impulse response functions. The qualitative behavior of entry, vacancies, market tightness, and unemployment is the same as in the benchmark: entry responds most on impact and then slowly recovers, whereas vacancies, tightness, and unemployment respond sluggishly to the shock reaching their peak response many periods after the shock. The difference between this calibration and the benchmark is in the magnitude and speed of the responses: vacancies, tightness, and unemployment peak at a level more than twice that of the benchmark and are less sluggish. In particular, vacancies peak after about 2 years.

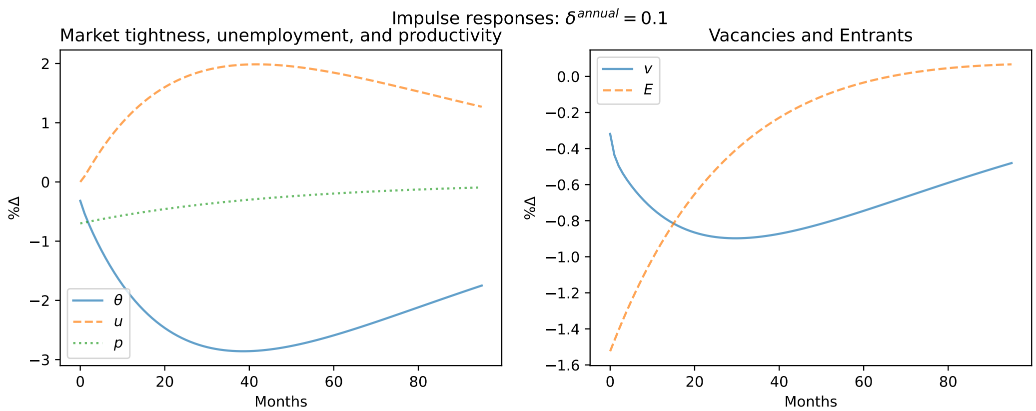

Figure 4: Impulse response functions to a one standard deviation negative technology shock for model with $\delta = 0.0087$.

Figure 5 graphs the impulse response functions generated by the model when all job losses are calibrated to be due to destruction, i.e. $s = 0$ and $\delta = 0.0342$, which corresponds to the calibration in Coles and Moghaddasi Kelishomi (2018). The general pattern from the previous two figures is present: following a productivity shock, the model predicts a sluggish response of vacancies, the market tightness, and entry. The extent of this sluggishness and the peak response in the vacancies and market tightness, however, are tightly linked to the calibrated value of the destruction shock: the larger δ is, the bigger the peak response and the faster vacancies and the market tightness converge to the steady state. Evidently, the calibrated value of the destruction shock is a key determinant of the model's numerical predictions. A lower destruction rate implies that most vacancies in the pool are surviving business opportunities from previous periods. As a result, new entrants comprise a small fraction of the pool. Since this is the portion of vacancies that responds to aggregate conditions the reduction of vacancies is small on impact. At the same time both the flows in (entry and separations) and out of vacancies (matching and destruction) are a relatively small fraction of the overall pool, which contributes to a sluggish response of vacancies. Because the behavior of the market tightness and unemployment are directly determined by the behavior of vacancies their response is qualitatively the same. Figure 5 also highlights the second source of sluggishness: congestion in entry induces firms to smooth out their response to productivity shocks over several periods.

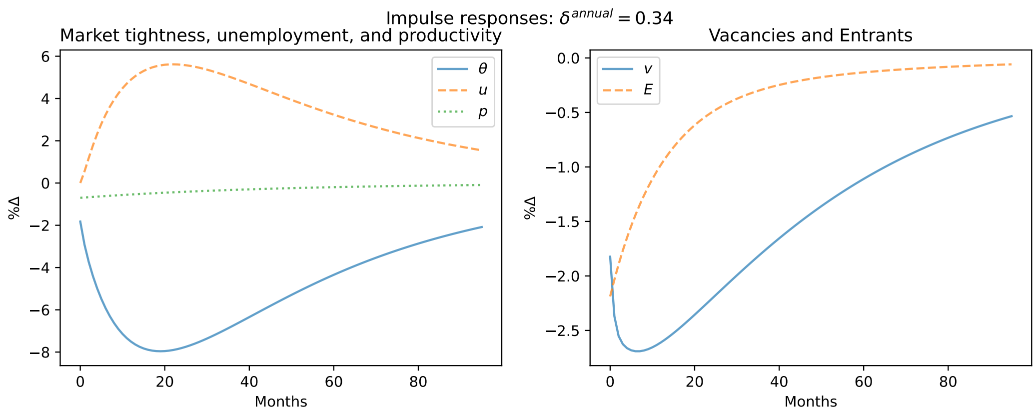

Figure 5: Impulse response functions to a one standard deviation negative technology shock for model with $\delta = 0.0342$.

When vacancies respond sluggishly to shocks, the labor-market variables co-move more strongly with past than with current levels of productivity. As a result, the absolute value of the cross-correlation between productivity and the labor market variables is lower. This phenomenon happens for two reasons.

First, a sluggish response in vacancies implies lower entry, so a higher fraction of the vacancy pool are vacancies that were created in previous periods. In those previous periods firms were looking at past levels of productivity to make their entry decisions, so those vacancies are correlated to these past productivity values. Second, a slower response in vacancies is associated with lower destruction rates. This means that a greater number of vacancies in the pool were established in prior periods, increasing the average age of the vacancy pool. As a result, the correlation between labor market variables and current productivity is weaker when compared to their correlation with past productivity.

Table 2 highlights this point by presenting the cross-correlation of productivity with

		Corr(X,p)				
Variable X	Data	Benchmark ($\delta = 0.0051$)	10% Destruction Rate ($\delta = 0.00874$)	SS ($\delta = 0.0181$)	FR/MS ($\delta = 0.0222$)	CM ($\delta = 0.0340$)
u	-0.43	-0.40	-0.49	-0.62	-0.66	-0.72
θ	0.44	0.51	0.58	0.73	0.77	0.84
v	0.43	0.73	0.77	0.89	0.92	0.95

Table 2: Contemporaneous correlations with productivity for different specifications of the destruction rate δ. The remaining parameters are recalibrated. Moments are based on quarterly averages of $150,000$ monthly observations. Productivity p is defined as output per hour from the Bureau of Labor Statistics and is available through FRED code PRS85006163. Each observable series ranges from 1951M1-2003M12 and is logged and HP-filtered with smoothing parameter $\lambda = 10^5$.

vacancies, unemployment, and the market tightness. The second column shows the model-predicted moments in the benchmark calibration and the third column reports the moments when the destruction shock is calibrated to match the empirically observed exit rate for firms. Column 4, referred to as SS, sets $\delta = 0.018$, which matches the calibration in Shao and Silos (2013). Column 5, referred to as FR/MS, matches the destruction rates used in Mercan and Schoefer (2020) and Fujita and Ramey (2007).[11] The final column, referred to as CM, follows Coles and Moghaddasi Kelishomi (2018) in calibrating the destruction rate to account for all separations.

The benchmark calibration in Column 2 is able to reproduce the mild cross-correlation of the labor market variables with productivity. For unemployment and the market tightness the correlation is slightly lower than that in the data, whereas for vacancies it is slightly higher. Turning to Column 3 and a destruction shock calibrated to match the data on firm exit, the model is able to replicate a mild cross-correlation for unemployment and the market tightness as well.

The correlation between vacancies and productivity is somewhat high at 0.69, yet still

[11] To be precise, Fujita and Ramey (2007) calibrate $\delta = 0.021$ but this small difference does not change the simulated moments in a meaningful way, so we group the two calibrations together.

significantly lower than what the baseline DMP model predicts and the values implied by models in the existing literature. In particular, for the last three columns vacancies are strongly pro-cyclical. A similar pattern emerges for unemployment and market tightness: when we calibrate the destruction shock to values from the existing literature, the cross-correlation is much higher than that in the data. Of course, the correlation does not approach unity even in the case when all separations are due to job destruction (Column CM), as the model still features the vacancy smoothing mechanism from Fujita and Ramey (2007). It is worth noting that, for low δ, the model is able to reproduce the mild cross-correlation of the labor variables with productivity while maintaining a strong correlation between the labor variables themselves. Specifically, in the benchmark calibration $\text{Corr}(u, v) = -0.91$ and $\text{Corr}(u, \theta) = -0.99$.

The main mechanism in our model, which breaks the near-perfect correlation between unemployment and productivity, also affects the volatility of unemployment. Unsurprisingly, a higher calibrated δ leads to higher volatility of unemployment. This is the case because higher destruction rates imply that the pool of vacancies is more responsive to productivity shocks. This, in turn, leads to higher volatility in the job-finding rate and ultimately unemployment.

The main focus of our paper is not matching unemployment volatility. However, matching the mild cyclicality of unemployment is compatible with generating empirically plausible volatility. Table 3 fixes $b = 0.95$ and varies ξ in the set $\{0.25, 0.5, 1.0, 5.0, 10, 15\}$. For each value, it reports the volatility of u and its correlation with productivity. This specification allows us to analyze any potential trade-off between generating mild $u - v$ correlation and unemployment volatility without altering the environment.

Under the benchmark value $\xi = 1$ the volatility is just over a third of that in the data. However, increasing the value of the elasticity allows the model to match the volatility of unemployment without drastically changing the magnitude of its correlation with productivity. For example, under $\xi = 5$, $\text{Corr}(u, p) = -0.5$ and $\text{SD}(u) = 0.21$.

We set ξ to unity in our benchmark because it is the value used in Fujita and Ramey (2007) and one of the values considered in Coles and Moghaddasi Kelishomi (2018). However, the literature has considered larger values of ξ as well: Haefke and Reiter (2020) use 15.878

for their benchmark and Qiu (2023) sets $\xi = 10.7$. Under these alternative values the model can generate more than enough amplification in unemployment to address the Shimer puzzle, yet the correlation between u and p remains mild as in the data. It should be noted, however, that the literature has also considered smaller values of the elasticity parameter. For example, Coles and Moghaddasi Kelishomi (2018) also consider a value of $\xi = 0.265$ and Potter (2022) calibrates the value to 0.01. These values inform the choice set of Table 3. Unsurprisingly, when ξ is smaller, there is little volatility in unemployment as congestion costs respond much more to changes in the number of entrants. Thus, Table 3 serves two purposes: it shows that (i) one can generate enough amplification in unemployment and at the same time match the mild correlation between unemployment and productivity; (ii) the model generates mild correlation between u and p for a large range of values for ξ.

ξ	Corr(u, p)	SD(u)
0.25	-0.20	0.034
0.50	-0.26	0.057
1.00	-0.41	0.07
5.00	-0.50	0.21
10.00	-0.54	0.30
15.00	-0.57	0.34

Table 3: Alternative values of ξ. The parameter $b = 0.95$ in each specification. The remaining parameters are recalibrated. Moments are based on quarterly averages of 150,000 monthly observations after applying logs and the HP filter with smoothing parameter $\lambda = 10^5$.

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3.3. Dynamic Correlations

A more comprehensive way to examine the time-series properties of the mechanism is via dynamic correlations. As we have stressed, when the destruction rate is low, the stock of vacancies is relatively more skewed towards vacancies created in past periods and therefore correlates more closely with past values of productivity. We observe this pattern for both the benchmark calibration and the calibration with 10% annual destruction rate: the dynamic correlations hold steady for four quarters. In contrast, for higher values of δ taken from the previous literature, where the correlation between vacancies and productivity diminishes with increasing lags. Table 4 shows the results for the different calibrated values of δ.[12]

		Corr(v_t, p_{t-i})			
Lagged Productivity	Benchmark ($\delta = 0.0051$)	10% destruction rate ($\delta = 0.00874$)	SS ($\delta = 0.0342$)	FR/MS ($\delta = 0.0222$)	CM ($\delta = 0.018$)
p_{t-1}	0.78	0.82	0.91	0.93	0.92
p_{t-2}	0.79	0.83	0.89	0.89	0.84
p_{t-3}	0.80	0.83	0.85	0.84	0.75
p_{t-4}	0.79	0.82	0.80	0.78	0.67

Table 4: Dynamic correlations. Each lag is at quarterly frequency. See description for Table 2.

Figure 6 plots the dynamic correlations at a monthly frequency for a more granular analysis.[13] For conciseness we focus on the benchmark calibration and the one in which the destruction rate is calibrated as by Fujita and Ramey (2007). The first panel of the figure shows the dynamic correlations between unemployment and productivity. Here the benchmark calibration better matches both the correlations at various frequencies. The curves corresponding to the two calibrations confirm our results from Table 4: the correlation peaks in the current period and peters off monotonically as lags increase for the calibration with $\delta = 0.0222$, whereas the peak correlation under the benchmark calibration is when productivity is lagged several months. Importantly, this is the behavior of the data as well: vacancies are more strongly correlated with past levels of productivity than current ones. Lastly, the third panel shows that the two calibrations produce comparable correlations between unemployment and vacancies.

[12] We only show the correlations for vacancies, but those for unemployment and the market tightness follow a similar pattern.

[13] We solve the model and simulate artificial data at a monthly frequency, as before, but now also apply the HP filter and compute moments at a monthly frequency.

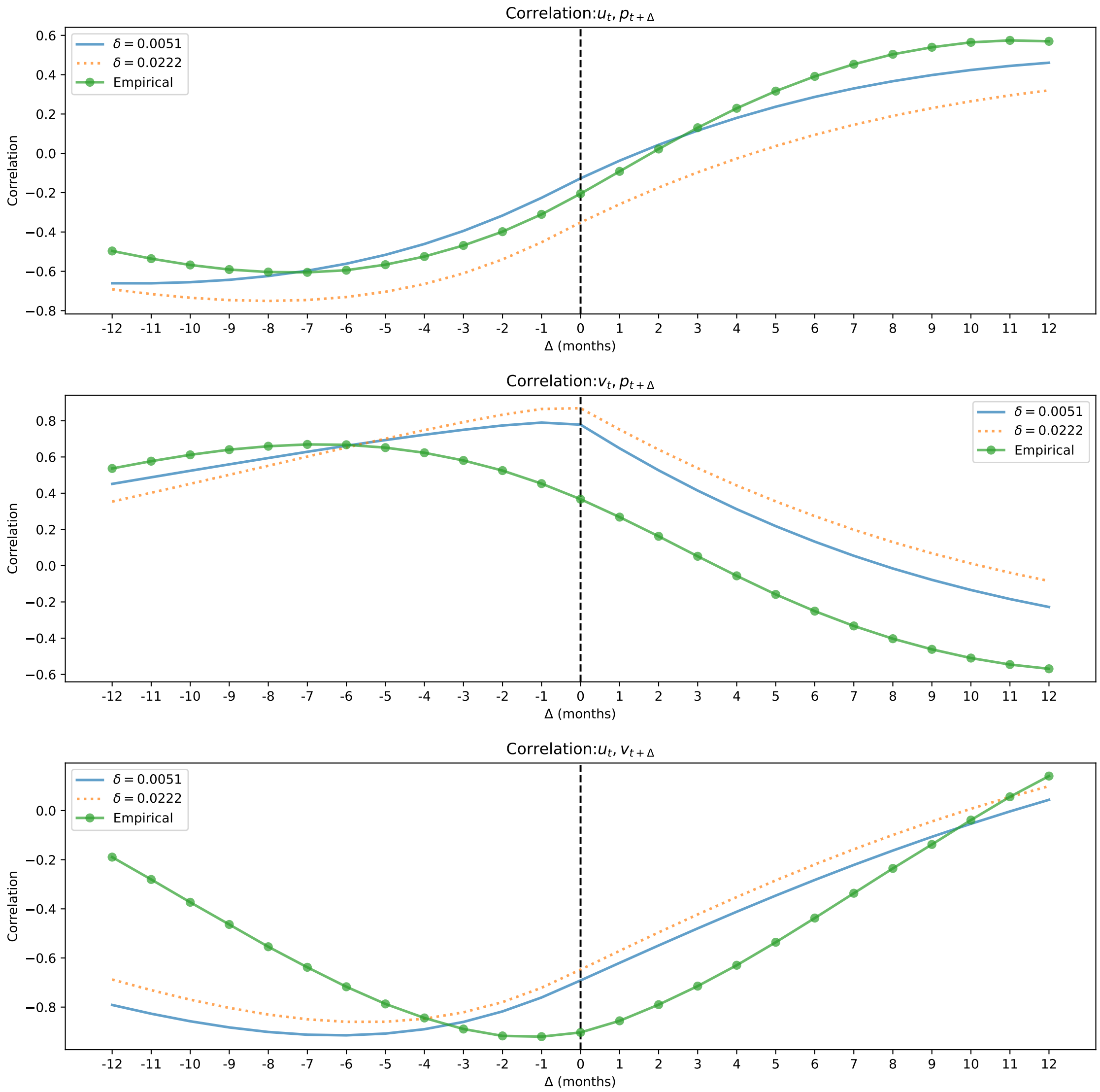

Figure 6: Dynamic correlations. The horizontal axis in each period depicts the time-shift Δ, measured in months, the vertical axis the correlation coefficient. For each pairwise combination of variables, we consider the data alongside the model calibrated under $\delta = 0.0051$ and $\delta = 0.0222$. Correlations are based on a sample of $100,000$ monthly observations. Both the data and model series are logged and HP-filtered with smoothing parameter $\lambda = 10^5$. The data range is 1951M1-2003M12.

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4. Conclusion

The Diamond-Mortensen-Pissarides framework has been extensively used to analyze labor market dynamics, with several extensions of the baseline model being able to reproduce the relative volatility of unemployment, vacancies, and market tightness. Yet the baseline model cannot reproduce the empirical mild correlation between productivity and labor market variables. We show that an extension with sunk vacancy creation costs and congestion in entry can help the model reproduce these empirical moments and also fit dynamic correlations reasonably well. The model can achieve these features with a technology shock as its only source of aggregate fluctuations.

The model eliminates the near-perfect correlation between the labor market variables and productivity for two reasons. First, some vacancies are created by new entrants whereas others originate from previous periods. As firms have no incentive to voluntarily destroy their vacancies, only the newly formed vacancies respond to a technology shock. That is, only a fraction of vacancies are determined by the current macroeconomic conditions, whereas the remaining fraction reflects past levels of productivity. The smaller the destruction shock, the smaller the pool of new entrants, and ultimately the less correlated vacancies are with current productivity.

Second, congestion in entry induces firms to smooth out entry in response to a shock, which serves to further reduce the correlation between vacancies and productivity. Our analysis shows that calibrating the economy with a destruction rate that matches (i) the micro-level evidence on product destruction and firm exits or (ii) the commonly used values in the growth literature allows the model to reproduce the mild cross-correlation between vacancies, unemployment, and the market tightness on the one hand and productivity on the other. The model achieves this feature while retaining the strong correlation between the labor market variables themselves.

Future research can enhance the understanding of the dynamics between businesses, product lines, and the labor market by explicitly incorporating time-series data and allowing for imperfect substitutability between product lines, as suggested by Bilbiie et al. (2012).

Appendix A. Derivations

Appendix A.1. Job creation condition

First, write the value of a vacancy as

$$Q_t = -\gamma + \beta(1-\delta)\mathbb{E}_t[q(\theta_t)(J_{t+1} - Q_{t+1}) + Q_{t+1}] \tag{A.1}$$

so that

$$\gamma + Q_t - \beta(1-\delta)\mathbb{E}_t Q_{t+1} = \beta(1-\delta)\mathbb{E}_t q(\theta_t)(J_{t+1} - Q_{t+1})$$

Use the variable $K_t = Q_t - \beta(1-\delta)\mathbb{E}_t Q_{t+1}$ and divide by $q(\theta_t)$ to find that

$$\frac{\gamma + K_t}{q(\theta_t)} = \beta(1-\delta)\mathbb{E}_t(J_{t+1} - Q_{t+1}) \tag{A.2}$$

Now, write the value of a filled job as

$$J_t = p_t - w_t + \beta(1-\delta)[J_{t+1} + s(Q_{t+1} - J_{t+1})] \tag{A.3}$$

Subtracting (A.3) from (A.1) we obtain

$$J_t - Q_t = p_t - w_t + \gamma + \beta(1-\delta)\mathbb{E}_t\left[(1 - s - q(\theta_t))(J_{t+1} - Q_{t+1})\right]$$

Forward (A.3), plug into (A.2), and rearrange to obtain the Euler equation.

Appendix A.2. Steady state

Appendix A.2.1. Derivation of basic form

Each steady state condition follows trivially from the general equilibrium conditions except (16):

$$e = \delta(v + 1 - u)$$

To see this, first rearrange the steady-state condition for vacancies as

$$v = \frac{s(1-\delta)(1-u) + e}{\delta + (1-\delta)q(\theta)}$$

Then note that

$$u = \frac{v}{\theta} = \frac{s(1-\delta)(1-u) + e}{\delta\theta + (1-\delta)f(\theta)}$$

Combining this with the steady-state Beveridge curve (15), we have

$$\frac{s(1-\delta)(1-u)+e}{\delta\theta+(1-\delta)f(\theta)} = \frac{\tau}{\tau+(1-\delta)f(\theta)}$$

Rearrange for e to find

$$\begin{aligned} e &= \delta v + (1-u)(\tau + s(1-\delta)) \\ &= \delta v + \delta(1-u) \\ &= \delta(v+1-u) \end{aligned}$$

Appendix A.2.2. Further properties

We can use (16) and (15) to derive the number of entrants as a ratio of θ:

$$e = \frac{\delta(\tau+(1-\delta)q(\theta))}{(1-\delta)q(\theta)}$$

As $\delta \to 0$, $e \to 0$. Moreover, as $\theta \to 0$, $q(\theta) \to 1$, so that $e \to \frac{\delta}{1-\delta}(\tau+1-\delta)$. As $\theta \to \infty$, $q(\theta) \to 0$, and $e \to \infty$. Thus, we can express K as a function of θ:

$$K = K[e(\theta)] = \left(\frac{\delta(\tau+(1-\delta)q(\theta))}{F(1-\delta)q(\theta)}\right)^{1/\xi} \frac{r+\delta}{1+r} \tag{A.4}$$

Note that, in contrast to γ, K rises with θ. The average hiring cost arising from investment in a product line is $K/q(\theta)$ therefore rises both directly from K and also because of shorter duration of a vacancy.

Now, let us re-examine the job creation condition:

$$\frac{\gamma+K(\theta)}{q(\theta)} = (1-\delta)\frac{(1-\alpha)(p-b-K(\theta))}{r+\tau+\alpha f(\theta)}$$

where we view $K(\theta)$ implicitly as a function of θ via (A.4). Note that the left-hand side $g_l(\theta)$ is increasing in θ and the right-hand $g_r(\theta)$ side is decreasing. Thus, equality, if it occurs, can occur only once. Since each side is a continuous function of θ, to show existence it suffices to find θ^* such that $g_l(\theta^*) < g_h(\theta^*)$ and θ^{**} such that $g_l(\theta^{**}) > g_h(\theta^{**})$.

Noting that $K(\theta) \to \infty$ as $\theta \to \infty$, we note that $g_r(\theta) \geq 0$, and we can choose θ^{**} to be the value such that $K(\theta^{**}) = p - b$.

Now choose $\theta^* = 0$, the lowest possible value of θ. We need to show that

$$\gamma + K(0) < (1-\delta)(1-\alpha)\frac{p-b-K(0)}{r+\tau} \Leftrightarrow$$

$$\gamma(r+\tau) + K(0)[r+\tau+(1-\delta)(1-\alpha)] < (1-\delta)(1-\alpha)(p-b)$$

using $q(0) = 1$ and $f(0) = 0$. Substitute for $K(0)$ and rearrange the condition as

$$\left(\frac{\delta}{1-\delta}\frac{\tau+1-\delta}{F}\right)^{1/\xi}\frac{r+\delta}{1+r} < \frac{(1-\delta)(1-\alpha)(p-b)-\gamma(r+\tau)}{r+\tau+(1-\delta)(1-\alpha)}$$

Appendix A.3. Elasticity of market tightness with respect to productivity

We examine the steady-state elasticity of market tightness with respect to productivity. First, apply logs to the steady state job creation condition (13):

$$\log\left(\frac{\gamma+K}{q(\theta)}\right) = \log\left(\frac{1-\delta}{r+\tau}\right) + \log(p-w-K)$$

Now differentiate with respect to $\log\theta$, treating w and K as constant. We obtain

$$-\frac{1}{\frac{\gamma}{q(\theta)}}\frac{\gamma+K}{q(\theta)^2}\frac{\partial q}{\partial \log\theta}\epsilon_{\theta,p} = \frac{1}{p-w-K}$$

$$\frac{1}{\frac{\gamma}{q(\theta)}}\frac{\gamma+K}{q(\theta)}\eta_L\epsilon_{\theta,p} = \frac{1}{p-w-K}$$

Rearrange as

$$\epsilon_{\theta,p} = \frac{1}{\eta_L(p-w-K)}$$

compared to $(1/(\eta_L(p-w)))$ in the baseline model. The direct effect of K is thus to increase amplification. However, the congestion channel dampens amplification. The congestion channel dissipates with higher ξ, disappearing as $\xi \to \infty$.

Appendix B. Alternate parameterization

We consider an equivalent microfoundation of the entry friction. Assume a firm can develop a product line at sunk entry cost ke_t^{ϕ}, so that the entry cost increases in the number of entrants. The value of a vacant firm with a product line is thus $Q_t = ke_t^{\phi}$. The flow entry cost becomes

$$K_t = k\mathbb{E}_t\left(e_t^{\phi} - \beta(1-\delta)e_{t+1}^{\phi}\right)$$

The mapping between (F,ξ) in the original parameterization and (k,ϕ) is $k = 1/F^{1/\xi}$ and $\phi = 1/\xi$.

This formulation is particularly useful for nesting the baseline DMP model. We recover that model by letting $\delta \to 0$ and $k \to 0$.

Appendix C. Numerical Algorithm

The algorithm for computing the dynamic stochastic equilibrium is an Euler-equation based method described in detail in Coleman et al. (2021). The unknown policy functions are approximated using complete quadratic monomials in terms of the states with coefficients Θ. There is one exogenous state variable p_t and the two endogenous state variables: unemployment u_t and predetermined vacancies $v_{pret,t} = (1-\delta)[(1-q(\theta_{t-1}))v_{t-1} + s(1-u_{t-1})]$. The aggregate state space is thus $\mathcal{S}_t = (u_t, v_{pret,t}, p_t)$. We use a quasi-random grid (Sobol) on a fixed hypercube to discretize the state space. We approximate the flow entry cost K_t and entrants $\mathbb{E}_t e_{t+1}^{1/\xi}$.

The steady state is useful as a precursor to computing the stochastic equilibrium for at least three reason: (1) it can be used to initialize the unknown functions at the steady state, (2) it is essential to express impulse responses in percentage deviations from steady state, and (3) it can be used to appropriately set bounds of the endogenous state variables.

1. Step 1: Initialization
   (a) Choose (u_t, F_t, p_t) and T
   (b) Choose approximating functions $H \approx \hat{H}(;\Theta)$
   (c) Make initial guesses on Θ: set equal to steady-state values
   (d) Choose integration nodes $\{\varepsilon_{x,j}\}_{j=1}^{J}$ and weights $\{\omega_j\}_{j=1}^{J}$
   (e) Construct a grid $\Gamma = \{u_m, v_{pret,m}, p_m\}_{m=1}^{M} \equiv \{X_m\}_{m=1}^{M}$
   (f) Choose termination criterion $crit = 1e-6$
2. Step 2: Computation of a solution for H
   (a) At iteration i, for $m = 1, \ldots, M$, compute
      - $K_m = \hat{H}_1(X_m), \mathbb{E}e_m^{'1/\xi} = \hat{H}_2(X_m)$
      - $e_m = [K_m F^{1/\xi} + \beta(1-\delta)\mathbb{E}e_m^{'1/\xi}]^{\xi}$
      - $v_m = v_{pret,m} + e_m$
      - $\theta_m = v_m/u_m$
      - $q_m = q(\theta_m)$
      - $f_m = \theta_m q_m$

- Update stocks

$$u'_m = (1 - (1 - \delta) f_m) u_m + \tau(1 - u_m)$$
$$v'_{pret,m} = (1 - \delta)((1 - q_m) v_m + s(1 - u_m))$$

- If necessary, constrain u'_m and $v_{pret,m}{}'$ to lie within admissible bounds
- Update state vector with future productivity nodes:

$$X_{mj} = (u'_m, v'_{pret,m}, p'_{mj}) \quad \forall j$$

- Interpolation of function

$$H'_{mj} = \hat{H}(X'_{mj}; \Theta)$$

- Repeat steps above to get $x'_{mj}, \theta'_{mj}, e'_{mj}$
- Numerical integration

$$EJ_m = \beta \sum_{j=1}^{J} \omega_j \left[(1 - \alpha)(x'_{mj} - b - K'_{mj}) - \alpha \theta'_{mj}(\gamma + K'_{mj})/(1 - \delta) + (1 - s) \frac{\gamma + K'_{mj}}{q'_{mj}} \right]$$

$$\hat{H}_{1,m} = q_m EJ_m - \gamma$$
$$\hat{H}_{2,m} = \beta \sum_{j=1}^{J} \omega_j e_{mj}^{'1/\xi}$$

(b) Find b that solves the system in (2a)

- Use ordinary least squares

$$\hat{\Theta}^g \equiv \arg\min \sum_{m=1}^{M} ||\hat{H}_m - \hat{H}(X_m; \Theta)||$$

- Dampening: weight η on new coefficients

$$\Theta^{(i+1)} = (1 - \eta)\Theta^{(i)} + \eta \hat{\Theta}^g$$

- Check for convergence: end Step 2 if

$$\frac{1}{M} \left\{ \sum_{m=1}^{M} |\frac{(H_m)^{i+1} - (H_m)^i}{(H_m)^i}|, \right\} < crit$$

(c) Iterate on Step 2 until convergence

The coefficients Θ give us approximate solutions to the policy functions. With the policy functions, it is straightforward to simulate data, construct moments, and generate impulse responses.

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